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ON THE EXISTENCE OF WEIERSTRASS POINTS WITH A CERTAIN SEMIGROUP GENERATED BY 4 ELEMENTS

By

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Introduction

Let X be a smooth, proper 1-dimensional algebraic variety (of genus$\geqq 2$) over an algebraically closed field k of characteristic 0, and let P be a point of X. Then a positive integer ν is called a *gap* at P if $h^0(X, \mathcal{O}_X((\nu-1)P))=h^0(X, \mathcal{O}_X(\nu P))$, and G_P denotes the set of gaps at P. If we denote by $\boldsymbol{N}$ and H_P respectively the additive semigroup of non-negative integers and the complement of G_P in $\boldsymbol{N}$, then H_P is a semigroup. A subsemigroup H of $\boldsymbol{N}$ whose complement is finite is called a *numerical semigroup*. The following problem is fundamental and is a long-standing problem.

Is there a pair (X, P) *with* X *a smooth, proper* 1-*dimensional algebraic variety over* k *and* P *its point, such that* $H=H_P$?

Using the deformation theory on algebraic varieties with $\boldsymbol{G}_m$-action, Pinkham [7] constructed a moduli space $\mathcal{M}_H$ which classifies the set of isomorphic classes of pairs (X, P) consisting of a smooth, proper 1-dimensional algebraic variety X together with its point P such that $H_P=H$. But he did not claim that $\mathcal{M}_H$ is non-empty. Using the Pinkham's construction of $\mathcal{M}_H$, some mathematicians showed that for some H, $\mathcal{M}_H$ is non-empty. To state their results we prepare some notation. Let $M(H)=\{a_1, \cdots, a_n\}$ be the minimal set of generators for the semigroup H, which is uniquely determined by H. I_H denotes the kernel of the k-algebra homomorphism $\varphi: k[X]=k[X_1, \cdots, X_n]\to k[t]$ defined by $\varphi(X_i)=t^{a_i}$ where $k[X]$ and $k[t]$ are polynomial rings over k, and $\mu(H)$ denotes the least number of generators for the ideal I_H. When we set $C_H=\mathrm{Spec}\, k[X]/I_H$, we denote by $T^1_{C_H}=\bigoplus_{l\in Z} T^1_{C_H}(l)$ the k-vector space of first order deformations of C_H with a natural graded structure. Moreover, $g(H)$ and $C(H)$ denote the cardinal number of the set $\boldsymbol{N}-H$ and the least integer c with $c+\boldsymbol{N}\subseteqq H$, respectively. Then $\mathcal{M}_H$ is non-empty in the following cases:

1) H is a complete intersection, i.e., $\mu(H)=n-1$,
2) H is a special almost complete intersection (Waldi [10]),

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