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COUNTABLE-POINTS COMPACTIFICATIONS OF PRODUCT SPACES

By

Takao HOSHINA

§ 1. Introduction.

Throughout this paper all spaces are assumed to be completely regular and T_1. A compactification αX of a space X is said to be a countable-points compactification (abbreviated to CCF) if the cardinality of the remainder $\alpha X-X$ is at most countable. Every locally compact space or, by L. Zippin [7], every rim-compact, Čech-complete separable metrizable space has a CCF. Thus, the problem to characterize those spaces which have a CCF was raised by K. Morita in [4], and for the case of metric spaces it was solved by the author [5].

In the present paper we consider the above problem on product spaces. Indeed, even for the case of a separable metrizable space X with a CCF and a compact space Y the product space $X\times Y$ does not have a CCF in general. More precisely, we shall establish the following theorems.

THEOREM 1. *Let X be a space having a CCF, and Y a zero-dimensional compact metrizable space. Then $X\times Y$ has also a CCF.*

THEOREM 2. *Let X be a paracompact space and Y a compact space. Then $X\times Y$ has a CCF iff X is locally compact or X has a CCF and Y is zero-dimensional and metrizable.*

Theorems 1 and 2 will yield further the following theorem which characterizes a product space of paracompact spaces to have a CCF.

THEOREM 3. *Let X and Y be paracompact spaces. Then $X\times Y$ has a CCF iff one of the following three conditions is satisfied:*

(a) *X and Y are both locally compact;*

(b) *one of X and Y is zero-dimensional, locally compact, separable metrizable, and the other has a CCF.*

(c) *X and Y are both zero-dimensional, Čech-complete separable metrizable.*

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