TSUKUBA J. MATH.
Vol.6 No. 2 (1982). 201–213

ON REFLECTION PRINCIPLES

By

Akito TSUBOI

Introduction.

In this paper, we shall consider various forms of reflection principles for 1-st order theories containing arithmetic. If a 1-st order theory T contains arithmetic, we can express various notions concerning T within T itself, by using the coding method developed by K. Gödel. K. Gödel assigned each formula ϕ in the language of T a number $\ulcorner\phi\urcorner$ (the Gödel number of ϕ), but our method is slightly different.

We assume that variables, individual constants, relation symbols and function symbols are numbers, and logical symbols ($*_\wedge*$, $*^\vee*$, $\neg*$, $*\rightarrow*$, $\forall**$, $\exists**$) are operations on numbers. Under these assumptions, a formula ϕ itself is a number and a theory T, which is a set of sentences, can be conceived as a subset of ω (the set of natural numbers). Let S be a theory and A a subset of ω. We say a formula $\alpha(x)$ in the language of S numerates A in S if, for any $n\in\omega$,

$$n\in A \text{ iff } S \text{ proves } \alpha(\bar{n}),$$

where $\bar{n}$ denotes the n-th numeral, i.e., the term of S which expresses the number n. In this case, we call this α a numeration of A in S. If α numerates A in S and $\neg\alpha$ numerates $\omega\backslash A$ in S, we say α binumerates A in S, and α is called a binumeration of A in S.

Let $A=\{n_1, \cdots, n_m\}$ be a subset of ω. Then $[A]$ denotes the formula $x=\bar{n}_1{}^\vee\cdots{}^\vee x=\bar{n}_m$. Clearly $[A]$ binumerates A in any theory S which contains arithmetic.

If a binumeration τ of a theory T in a theory S is given, we can construct a provability formula $Pr_\tau(x)$ whose intuitive meaning is that a formula x is provable in T. The reader should note that this Pr_τ cannot be uniquely determined by T, but is determined by τ. (The explicit definition of Pr_τ can be found in p. 59 of [1].)

Using this Pr_τ, we define the τ-reflection principle $Rfn(\tau)$ and the τ-reflection principle $Rfn_A(\tau)$ based on A:

Received October 9, 1981. Revised March 26, 1982.