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# WEAKLY DIVISIBLE AND DIVISIBLE MODULES

(Dedicated to Prof. K. Murata for his sixtieth birthday)

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## 1. Introduction.

Recently in his paper [5], M. Sato introduced the concept of weakly divisible modules and called a preradical $t$ pseudo-cohereditary if every weakly divisible module is divisible. On the other hand, J. Jirásko [2] called a preradical $t$ pseudo-cohereditary if every homomorphic image of $M/(M\cap t(E(M)))$ is torsionfree for each module $M$, where $E(M)$ denotes the injective hull of $M$. Now we call an exact squence $0\rightarrow A\xrightarrow{j} B$ of modules coindependent (resp. weakly coindependent) if $B=j(A)+t(B)$ (resp. $B/(j(A)+t(B))$ is torsion). By means of these notions, we give, in the first half of this paper, some characterizations of weakly divisible and divisible modules. In the latter half, we inquire into relations between two pseudo-cohereditarities in the sense of Sato and Jirásko.

## 2. Weakly divisible modules and coindependent sequences.

Throughout this note $R$ means a ring with identity and modules mean unitary left $R$-modules. We denote the category of left $R$-modules by $R$-mod. A subfunctor $t$ of the identity functor of $R$-mod is called a *preradical* of $R$-mod. It is called *idempotent* if $t(t(M))=t(M)$ and a *radical* if $t(M/t(M))=0$ for all modules $M$. Also, it is called *left exact* if $t(N)=t(M)\cap N$ and *cohereditary* if $t(M/N)=(t(M)+N)/N$ for all modules $M$ and submodules $N$. To each preradical $t$ of $R$-mod, we put

$$T(t)=\{M\in R\text{-mod}\mid t(M)=M\}\quad\text{and}\quad F(t)=\{M\in R\text{-mod}\mid t(M)=0\}\ .$$

In general, $T(t)$ is closed under homomorphic images and direct sums, while $F(t)$ is closed under submodules and direct products.

DEFINITION 2.1. For a preradical $t$, a module $M$ is called *weakly divisible* (resp. *divisible*) with respect to $t$ if the functor $\mathrm{Hom}_R(-, M)$ preserves the ex-

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