TSUKUBA J MATH.
Vol. 6 No. 2 (1982). 187—194

A BOOLEAN POWER AND A DIRECT PRODUCT OF ABELIAN GROUPS

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A group means an abelian group in this paper. A Boolean power and a direct product of groups consist of all global sections of groups in some Boolean extensions $V^{(B)}$. We shall study about a homomorphism h whose domain is a group consisting of all the global sections of a group in $V^{(B)}$. We investigate two cases: one of them is that the range of h is a slender group, which is related to a torsion-free group, and the other is that the range of h is an infinite direct sum, which is related to a torsion group. We extend a few theorems which have been obtained in [4] and [5]. As in [5], we not only extend theorems, but improve them and give a good standing point of view.

We refer the reader to [9] or [1], for a Boolean extension $V^{(B)}$. We shall use notations and terminologies in [5], [6] and [7]. Throughout this paper, $\boldsymbol{B}$ is a complete Boolean algebra and $\mathcal{F}$ is the set of all countably complete maximal filters on $\boldsymbol{B}$. We do not mention these any more. $\check{x}$ is the element of $V^{(B)}$ such that dom $\check{x}=\{\check{y}\,;\,y\in x\}$ and range $x\subseteq\{\mathbf{1}\}$. As noted in [5], "$\check{x}$" in [1] means our "$\check{x}$". $\hat{x}=\{y\,;\,[\![y\in x]\!]=\mathbf{1}$ and $y\in V^{(B)}\}$ for $x\in V^{(B)}$, where $V^{(B)}$ is separated. For $b\in\boldsymbol{B}$ and a group A in $V^{(B)}$, i.e. $[\![A$ is a group$]\!]=\mathbf{1}$, $\hat{A}^b$ is the subgroup of $\hat{A}$ such that $x\in\hat{A}^b$ iff $x\in\hat{A}$ and $-b\leqq[\![x=0]\!]$, where 0 is the unit of A. By this notation, $\hat{A}=\hat{A}^1$. For $x\in\hat{A}$, x^b is the element of $\hat{A}^b$ such that $b\leqq[\![x=x^b]\!]$.

1. A general setting about a complete Boolean algebra

Let $\Phi(b)$ be a property of $b\in\boldsymbol{B}$ which satisfies the following conditions:

(1) if $\{b_n\,;\,n\in N\}$ is a pairwise disjoint subset of $\boldsymbol{B}$, there exists k such that $\Phi(\bigvee_{n\geq k}b_n)$ and $\Phi(b_n)$ hold for each $n\geqq k$;

(2) if $b\wedge c=\mathbf{0}$, $\Phi(b)$ and $\Phi(c)$ hold, then $\Phi(b\vee c)$ holds.

Let S be the subset of $\boldsymbol{B}$ such that $b\in S$ iff $\Phi(b)$ does not hold and $c\wedge c'=\mathbf{0}$ implies $\Phi(c)$ or $\Phi(c')$ for any $c, c'\leqq b$.

Received February 18, 1982.
The author is partially supported by Grant-in-Aid for Encouragement of Young Scientist Project No. 56740087.