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CLASS GROUPS OF GROUP RINGS WHOSE COEFFICIENTS ARE ALGEBRAIC INTEGERS

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Let R be the ring of integers of an algebraic number field k. Let Λ be an R-order in a finite dimensional semisimple k-algebra A. We mean by the class group of Λ the class group defined by using locally free left Λ-modules and denote it by $C(\Lambda)$. We define $D(\Lambda)$ to be the kernel of the natural subjection $C(\Lambda)\rightarrow C(\Omega)$, where Ω is a maximal R-order in A containing Λ, and denote by $d(\Lambda)$ the order of $D(\Lambda)$. $C(\Omega)$ is isomorphic to a (narrow) ideal class group of the center of A, which is a product of the ideal class groups of algebraic number fields with modulus some real infinite primes. Hence, in a sense, we may concentrate on $D(\Lambda)$.

Let G be a finite group and let RG be the group ring of G with coefficients in R. Then RG can be regarded as an R-order in the semisimple k-algebra kG. We define $T(RG)$ to be the kernel of the natural surjection $C(RG)\rightarrow G(R)\oplus C(RG/(\Sigma_{G}))$, where $\Sigma_{G}=\sum_{g\in G}g\in RG$, and denote by $t(RG)$ the order of $T(RG)$. Then $T(RG)\cong\mathrm{Ker}\,(D(RG)\rightarrow D(RG/(\Sigma_{G})))$. Throughout this paper, C_{n} denotes the cyclic group of order n and p stands for a rational prime.

Much investigation has been done on $D(\boldsymbol{Z}G)$ and $T(\boldsymbol{Z}G)$ (cf. [8]), but the results seem to depend on the speciality of $\boldsymbol{Z}$.

The purpose of this paper is to study $D(RG)$ for the case where $R\neq\boldsymbol{Z}$. In §1 we give some basic results on $D(RG)$ and $T(RG)$. In §2∼§4 we assume that R is the ring of integers in a quadratic field. We first give some results on $D(RC_{p^{e}})$, and next examine the structure of $D(RC_{p})$.

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§1.

For a ring S, $U(S)$ denotes its unit group. For an abelian group A and a positive integer q, $A^{(q)}$ denotes the q-part of A and $A^{(q')}$ denotes the maximal

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