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ON PROJECTIVE COHEN-MACAULAYNESS OF A DEL PEZZO SURFACE EMBEDDED BY A COMPLETE LINEAR SYSTEM

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Let k be an algebraically closed field. We understand by a Del Pezzo surface X over k a non-singular rational surface on which the anti-canonical sheaf $-\omega_X$ is ample. We call the self-intersection number $d=\omega_X^2$ of ω_X the degree of X, then we get that $1\leqq d\leqq 9$. It is well known that X is isomorphic to $\boldsymbol{P}^1\times\boldsymbol{P}^1$, which has degree 8, or an image of $\boldsymbol{P}^2$ under a monoidal transformation with center the union of $r=9-d$ points which satisfies the following conditions:

(a) no three of them lie on a line;

(b) no six of them lie on a conic;

(c) there are no cubics which pass through seven of them and have a double point at the eighth point.

Conversely any surface described above is a Del Pezzo surface of the corresponding degree ([8, III, Theorem 1]). It is also well known that $-\omega_X$ is very ample when $d\geqq 3$ and that ample divisors on X of degree 3, which is a cubic surface, are very ample too. In this paper we will get that *ample divisors on X of degree $d\geqq 3$ are very ample and that ample divisors on X of degree* 2 [*resp.* 1] *other than* $-\omega_X$ [*resp.* $-\omega_X$ *nor* $-2\omega_X$] *are very ample.*

A closed subscheme V in $\boldsymbol{P}^N$ is said to be projectively Cohen-Macaulay if its affine cone is Cohen-Macaulay. It is equivalent to that $H^1(\boldsymbol{P}^N, \mathcal{J}_V(m))=0$ for every $m\in Z$ and $H^i(V, \mathcal{O}_V(m))=0$ for every $m\in Z$ and $0<i<\dim V$. In this paper, we will get that $\phi_{|D|}(X)$ is projectively Cohen-Macaulay for a very ample divisor D on X, where $\phi_{|D|}$ is the morphism from X to $\boldsymbol{P}^{\dim|D|}$ defined by the complete linear system $|D|$ of D. We also study the homogeneous ideal $I(D)=\mathrm{Ker}\left[S\Gamma(D)\longrightarrow\bigoplus_{n\geqq 0}\Gamma(nD)\right]$ defining $\phi_{|D|}(X)$. These results will be stated and proved in §3 and §5. The fourth section will be devoted to a study on $-n\omega_X$ of a Del Pezzo surface X of degree 1 or 2.

In §1 we will compute the dimension $h^i(D)$ of the i-th cohomology group $H^i(X, \mathcal{O}_X(D))$ of the invertible sheaf $\mathcal{O}_X(D)$ corresponding to a divisor D.

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