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ON THE CURVES OF GENUS g WITH AUTOMORPHISMS OF PRIME ORDER $2g+1$

By

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Introduction.

Let k be an algebraically closed field, and let C be a complete non-singular curve of genus $g\geqq 2$ defined over k. In [2], M. Homma showst hat if a prime number q is the order of an automorphism of C, then $q\leqq g+1$ or $q=2g+1$. He determines all C in the case of $q=2g+1$ as follows:

(i) If q is equal to the characteristic p of k, then C is birationally equivalent to the plane curve

$$y^{2}=x^{q}-x.$$

(ii) If q is not equal to p, then C is birationally equivalent to one of the following plane curves

$$y^{m-r}(y-1)^{r}=x^{q},\quad 1\leqq r<m\leqq g+1.$$

The case (ii) shows, in particular, there may be many isomorphy classes of curves of genus g which admit an automorphism of prime order $2g+1\neq p$. The aim of this paper is to classify these curves.

Fix a prime number $q\geqq 5$ different from p. For a pair of positive integer (r, s) such that any one of r, s and $r+s$ is coprime to q, let $C(r, s)$ be a non-singular model of the irreducible equation

$$y^{r}(y-1)^{s}=x^{q}$$

over k. Then the genus of $C(r, s)$ is $(q-1)/2$ and $C(r, s)$ has an automorphism of order q. In § 1, we shall give a basis of the space oı differentials of the first kind on $C(r, s)$, in forms suitable to our later use. In § 2, we shall give a condition under which $C(r, s)'s$ are isomorphic in terms of r and s. This is our main result. In particular, we see that the cardinality of the set of isomorphy classes is, $(q+5)/6$ if $q\equiv 1$ mod 3, and $(q+1)/6$ if $q\equiv 2$ mod 3. In § 3, we determine the order of the group of automorphisms of $C(r, s)$ in the case of characteristic zero.

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