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SPACES WITH A PROPERTY RELATED TO UNIFORMLY LOCAL FINITENESS

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Throughout this paper a space always means a topological space.

A collection $\mathcal{A}$ of subsets of a space X is said to be *uniformly locally finite* if there is a normal open cover $\mathcal{U}$ of X such that each member of $\mathcal{U}$ intersects only finitely many members of $\mathcal{A}$. If every locally finite collection of subsets of X is uniformly locally finite, then X is said to have *property* (U). These notions are defined in K. Morita [10], and it is pointed out there that every M-space or every strongly normal (=collectionwise normal and countably paracompact) space is a space with property (U), and such a space is expandable in the sense of L. L. Krajewski [7]. Hence for normal spaces property (U), expandability and strong normality all coincide with each other by a well-known theorem of M. Katětov [6], and so a question was posed by Morita [10] to find a condition which, together with expandability, is equivalent to property (U).

The purpose of this paper is to investigate spaces with property (U), mainly by defining a new notion of U-embedding which is a generalization of P-embedding; a subspace A of a space X is said to be *U-embedded* in X if every uniformly locally finite collection of subsets of A is uniformly locally finite also in X. In §1 we treat spaces having a property that every discrete collection of subsets is uniformly locally finite, which we call spaces with property $(U)^*$. By C. H. Dowker [1], collectionwise normal spaces are precisely those spaces any of whose closed set is P-embedded. Being motivated with this result we shall establish a theorem that a space X has property $(U)^*$ iff any closed set of X is U-embedded in X, and then it will be shown that a space has property (U) iff it has property $(U)^*$ and is a cb-space in the sense of J. Mack [9]; the latter is a quite analogue to a theorem of Krajewski [7] that a space is expandable iff it is discretely expandable and countably paracompact. In §2 we shall give another description of spaces with property (U), which is an answer to the question of Morita above, by defining spaces with weak property (U) that include all M'-spaces [5] and all extremally disconnected spaces.

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