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ON A RELATION BETWEEN THE TOTAL CURVATURE AND THE MEASURE OF RAYS

Dedicated to Professor I. Mogi on his 60th birthday

By

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§ 0. Introduction.

Let X be a 2-dimensional manifold, then we say that X is finitely connected if the fundamental group $\pi_1(X)$ is finitely generated. If X is noncompact and finitely connected, then it is homeomorphic to a compact surface with a finite number of points removed. Let M be a 2-dimensional finitely connected complete noncompact Riemannian manifold without boundary. The Euler characteristic of $M, \chi(M)$, equals the Euler characteristic of the associated compact surface minus the number of points removed. A geodesic $\gamma:[0, \infty)\rightarrow M$ is called a ray when any subarc of γ is the shortest connection between its end points. And all geodesics are assumed to be parametrized by arc length. Let T_pM be the tangent space of M at p and S_pM be the unit circle of T_pM centered at the origin. S_pM may be regarded as a standard unit circle S^1 from the Euclidean metric on T_pM. Hence we can consider the Riemannian measure on S_pM. Let $A(p)$ be the subset of S_pM consisting of vectors v in S_pM such that the geodesic $\gamma_v:[0, \infty)\rightarrow M, \gamma_v(t)=\exp_p tv$, is a ray, where $\exp_p$ is the exponential map of M.

Recently, Maeda has proved in [4] the following theorem with interest in a problem whether less curvedness of a Riemannian manifold in some sense implies the existence of rays on it in large quantities or not when the manifold is non-negatively curved;

THEOREM ([4]). Let M be a 2-dimensional complete Riemannian manifold with nonnegative Gaussian curvature $G\geqq 0$ diffeomorphic to a Euclidean plane. If $\int_M G\,dv<2\pi$, then for any point p in M such that $\#A(p)\geqq 2$, we have

$$\text{measure } A(p)\geqq 2\pi-\int_M G\,dv.$$

Here the total curvature $\int_M G\,dv$ of a noncompact Riemannian manifold M is by

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