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HOMEOMORPHISMS OF INFINITE-DIMENSIONAL FIBRE BUNDLES

By

Katsuro SAKAI

0. Introduction

Throughout in this paper, *all spaces are metrizable* and E denotes a locally convex linear metric space homeomorphic ($\cong$) to its own countably infinite product E^{ω} or the subspace E_f^{ω} of E^{ω} consisting of all elements whose almost all coordinates are zero.

A *manifold modeled on* E, briefly E-*manifold*, is a metric space M admitting an open cover by sets homeomorphic to open subsets of E. We assume the following:

Each E-*manifold has the same weight as* E.

Hence if E is separable, then E-manifolds means separable E-manifolds. It is well-known that *each connected* E-*manifold has the same weight as* E.

An E-*manifold bundle* is a locally trivial fibre bundle with fibre an E-manifold. An E-manifold bundle with fibre M is briefly called an M-*bundle*. Then an E-bundle is a locally trivial fibre bundle with fibre E. It is proved by T. A. Chapman [Ch_3] that *each* E-*bundle is trivial*, that is, bundle isomorphic a product bundle. In this paper (Section 3 and 4), we show that each E-manifold bundle over B can be embedded in the product bundle $B\times E$ as a closed or/and an open sub-bundle.

Let $p: X\to B$ be an E-manifold bundle. By Bundle Stability Theorem [Sa_2], $p: X\to B$ is bundle isomorphic to $p\circ\mathrm{proj}: X\times E\to B$. A subset K of X is said to be B-*preservingly* E-*deficient* if there exists a bundle homeomorphism $h: X\to X\times E$ such that $h(K)\subset X\times\{0\}$. From 5-2 in [Sa_2], we can require h to satisfy that $h(x)=(x, 0)$ for each $x\in K$. In this paper, we research several properties of B-preservingly E-deficient sets.

In Section 2, we show that a B-preservingly E-deficient locally closed set K is negligible in X, that is, $p|X\setminus K: X\setminus K\to B$ is also an E-manifold bundle which is bundle isomorphic to $p: X\to B$. And in Section 4, we show that if Y is a B-preservingly E-deficient closed set in X and $p|Y: Y\to B$ is also an E-manifold bundle, then Y is B-preservingly collared in X, that is, there is an open embedding

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