

## ON THE ADJUNCTION SPACES OF FREE $L$ -SPACES AND $M_1$ -SPACES

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A class of free  $L$ -spaces is defined by Nagami [7]. This class contains all Lašnev spaces and is contained in the class of  $M_1$ -spaces in the sense of Ceder [3]. In this paper, we consider the sum theorem of free  $L$ -spaces and the property of being  $M_1$ -spaces and free  $L$ -spaces of the adjunction spaces. The main results are as follows:

1. Let  $Z=X \cup Y$  be stratifiable, where  $X, Y$  are free  $L$ -spaces and  $X$  is a closed set of  $Z$  with a uniformly approaching anti-cover in  $Z$ . Then  $Z$  is a free  $L$ -space.
2. The adjunction space  $X \cup_f Y$  is a free  $L$ -space if  $X$  is an  $L$ -space in the sense of Nagami [6] and  $Y$  is a free  $L$ -space.
3. Let  $Z=X \cup Y$  be stratifiable, where  $X, Y$  are  $M_1$ -spaces and  $X$  is a closed set with a uniformly approaching anti-cover in  $Z$ . Then  $Z$  is an  $M_1$ -space.
4. The adjunction space  $Z=X \cup_f Y$  is an  $M_1$ -space if  $X$  is a free  $L$ -space and  $Y$  is an  $M_1$ -space.
5. Every closed set of a free  $L$ -space has a closure-preserving open neighborhood base.
6. The closed irreducible image of an  $M_1$ -space with  $\dim=0$  is also an  $M_1$ -space.

All spaces are assumed to be Hausdorff and mappings to be continuous and onto unless the contrary is stated explicitly.  $N$  always denotes the positive integers. As for undefined term, see Nagami [6] and [7], or [4].

A space  $X$  is called a *monotonically normal space* if the following (MN) is satisfied:

(MN) To each pair  $(H, K)$  of separated subsets of  $X$ , one can assign an open set  $U(H, K)$  in such a way that

- (i)  $H \subset U(H, K) \subset \overline{U(H, K)} \subset X - K$  and
- (ii) if  $(H', K')$  is a pair of separated sets having  $H \subset H'$  and  $K' \subset K$ , then  $U(H, K) \subset U(H', K')$ .

LEMMA 1 ([4, Lemma 3.1]). *Let  $X$  be a monotonically normal space,  $F$  a*