TSUKUBA J. MATH.
Vol. 6 No. 1 (1982). 1—8

COVERINGS OF GENERALIZED CHEVALLEY GROUPS ASSOCIATED WITH AFFINE LIE ALGEBRAS

By

Jun MORITA

R. Steinberg [21] has given a presentation of a simply connected Chevalley group (=the group of k-rational points of a split, semisimple, simply connected algebraic group defined over a field k) and has constructed the (homological) universal covering of the group. In this note, we will consider an analogy for a certain family of groups associated with affine Lie algebras.

1. Chevalley groups, Steinberg groups and the functor $K_2(\Phi, \cdot)$.

Let Φ be a reduced irreducible root system in a Euclidean space $\boldsymbol{R}^n$ with an inner product $(\cdot, \cdot)$ (cf. [4], [6]). We denote by Φ^+ (resp. Φ^-) the positive (resp. negative) root system of Φ with respect to a fixed simple root system $\Pi=\{\alpha_1, \cdots, \alpha_n\}$. We suppose that α_1 is a long root (for convenience' sake). Let α_{n+1} be the negative highest root of Φ. Set $a_{ij}=2(\alpha_i, \alpha_j)/(\alpha_j, \alpha_j)$ for each $i, j=1, 2, \cdots, n+1$. The matrices $A=(a_{ij})_{1\leq i, j\leq n}$ and $\tilde{A}=(a_{ij})_{1\leq i, j\leq n+1}$ are called a Cartan matrix of Φ and the affine Cartan matrix associated with A respectively (cf. [4], [5], [6]).

Let $G(\Phi, \cdot)$ be a Chevalley-Demazure group scheme of type Φ (cf. [1], [20]). For a commutative ring R, with 1, we call $G(\Phi, R)$ a Chevalley group over R. For each $\alpha\in\Phi$, there is a group isomorphism—"exponential map"—of the additive group of R into $G(\Phi, R): t\longmapsto x_\alpha(t)$. The elementary subgroup $E(\Phi, R)$ of $G(\Phi, R)$ is defined to be the subgroup generated by $x_\alpha(t)$ for all $\alpha\in\Phi$ and $t\in R$. We use the notation $G_1(\Phi, \cdot)$ and $E_1(\Phi, \cdot)$ (resp. $G_0(\Phi, \cdot)$ and $E_0(\Phi, \cdot)$) if $G(\Phi, \cdot)$ is simply connected (resp. of adjoint type). It is well-known that $G_1(\Phi, R)=E_1(\Phi, R)$ if R is a Euclidean domain (cf. [22, Theorem 18/Corollary 3]).

Let $St(\Phi, R)$ be the group generated by the symbols $\hat{x}_\alpha(t)$ for all $\alpha\in\Phi$ and $t\in R$ with the defining relations

(A) $\hat{x}_\alpha(s)\hat{x}_\alpha(t)=\hat{x}_\alpha(s+t)$,

(B) $[\hat{x}_\alpha(s), \hat{x}_\beta(t)]=\prod \hat{x}_{i\alpha+j\beta}(N_{\alpha,\beta,i,j}s^it^j)$,

(B)′ $\hat{w}_\alpha(u)\hat{x}_\alpha(t)\hat{w}_\alpha(-u)=\hat{x}_{-\alpha}(-u^{-2}t)$

for all $\alpha, \beta\in\Phi(\alpha+\beta\neq 0)$, $s, t\in R$ and $u\in R^*$, the units of R, where $\hat{w}_\alpha(u)=\hat{x}_\alpha(u)\hat{x}_{-\alpha}(-$

Received March 11, 1981.