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ON THE CAUCHY PROBLEM FOR A SEMI-LINEAR HYPERBOLIC SYSTEM AND ITS TRAVELING WAVE-LIKE SOLUTIONS

By

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Introduction

We consider the Cauchy problem of the following system of semi-linear partial differential equations for $u(x,t)$ and $v(x,t)$:

$$(1)\quad \begin{cases} \dfrac{\partial u}{\partial t}+\lambda\dfrac{\partial u}{\partial x}=-uv+g(u)\varepsilon\,, \\ \dfrac{\partial v}{\partial t}+\mu\dfrac{\partial v}{\partial x}=uv+h(v)\varepsilon, \quad (x,t)\in R\times R_+\,, \end{cases}$$

with the initial data

$$(2)\quad \begin{cases} u(x,0)=\phi(x)\,, \\ v(x,0)=\psi(x), \quad x\in R\,, \end{cases}$$

where $R=(-\infty,+\infty)$ and $R_+=(0,+\infty)$; $\lambda, \mu(\lambda\neq\mu)$ and ε are real constants; g and h are real-valued and real analytic functions at the origin with radii ρ_1 and ρ_2 respectively, that is to say

$$(3)\quad \begin{cases} g(u)=\sum_{k=0}^{\infty}a_k u^k, \quad h(v)=\sum_{k=0}^{\infty}b_k v^k\,; \\ \limsup\limits_{k\to\infty}\sqrt{|a_k|}=\dfrac{1}{\rho_1}, \quad \limsup\limits_{k\to\infty}\sqrt{|b_k|}=\dfrac{1}{\rho_2}\,. \end{cases}$$

Without loss of generality we may assume that $0<\rho_1\leqq\rho_2$, and we suppose that

$$(4)\qquad \phi(x), \psi(x)\geqq 0,\ x\in R\,;\ \phi(x), \psi(x)\in\mathfrak{B}^1(R)\,,$$

where by $\mathfrak{B}^1(R)$ we mean the function space of all real-valued C^1-functions which are bounded on R together with their first derivatives. From now on by $C^1(S)$ we mean the function space of all real-valued continously differentiable functions defined on S.

The system (1)-(2) has an ecological meaning when both g and h are some

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