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DEGREE OF THE STANDARD ISOMETRIC MINIMAL IMMERSIONS OF THE SYMMETRIC SPACES OF RANK ONE INTO SPHERES

Dedicated to Professor Isamu Mogi on his 60th birthday

By

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Introduction

Let $(M=G/K, g)$ be an irreducible symmetric space of compact type. We denote by Δ the Laplace-Beltrami operator of (M, g) acting on the space of C^∞ functions on M. We denote by λ_k the k-th eigen-value of Δ and by V^k the corresponding eigen-space. For each integer k, $k\geqq 1$, there exists an isometric minimal immersion x_k of M into the unit sphere in $R^{m(k)+1}$ defined by an orthonormal base of V^k, which we call the k-th standard isometric minimal immersion of M.

do Carmo and Wallach [2] showed that the k-th standard isometric minimal immersion of a sphere S^n has degree k. We showed that the k-th standard minimal immersion of a complex projective space CP^n, $n\geqq 2$, has degree $2k$ [5]. In these cases the degree of the immersion coincides with the algebraic degree of the homogeneous polynomials which define the immersion. In this note we determine the degree of the standard isometric minimal immersions of the other symmetric spaces of rank one into spheres.

THEOREM A. *Let x_k be the k-th standard isometric minimal immersion of a quaternion projective space QP^n, $n\geqq 2$. Then x_k has degree $2k$.*

Let $\pi: S^{4n+3}\to QP^n$ be the Hopf fibration, where we consider S^{4n+3} as the unit sphere in $Q^{n+1}=C^{2n+2}$. Then for each eigen-function f on QP^n which belongs to V^k, there exists a homogeneous harmonic polynomial P_f of type (k, k) on C^{2n+2} which induces f through π. So the degree of the immersion coincides with the algebraic degree of the homogeneous polynomials which define the immersion.

THEOREM B. *Let x_k be the k-th standard isometric minimal immersion of the Cayley projective plane $CayP^2$. Then x_k has degree $2k$.*

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