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SHAPE FIBRATIONS AND FIBER SHAPE EQUIVALENCES, II

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0. Introduction.

In [2] and [3], Coram and Duvall introduced the notion of approximate fibrations and they characterized this in terms of movability conditions for maps. Mardešić and Rushing [11] defined shape fibrations and showed that for compact ANR's, those agree with approximate fibrations. In [8], we defined fiber fundamental sequences and fiber shape equivalences.

In this paper, we show that fiber fundamental sequences have shape theoretic properties analogous to the homotopy theoretic properties of fiber maps. In particular, we prove the following:

(1) Let E and B be compacta and let B be an FAR. Then a map $p:E\to B$ is a shape fibration if and only if p is shape trivial.

(2) A proper map $p:E\to B$ between locally compact, separable metric ANR's is an approximate fibration if and only if p is locally shape trivial.

(3) Let $p:E\to B$ be a shape fibration from a compactum E to a connected compact ANR B and let $p':E'\to B$ be an approximate fibration between compact ANR's. Then a fiber map $f:E\to E'$ over B is a fiber shape equivalence over B if and only if for some $b_0\in B$, $f|p^{-1}(b_0):p^{-1}(b_0)\to p'^{-1}(b_0)$ is a shape equivalence.

It is assumed that all spaces are metrizable and all maps are continuous. If x and y are points of a metric space, $d(x,y)$ denotes the distance from x to y. For maps $f,g:X\to Y$ of compacta, $d(f,g)=\sup\{d(f(x),g(x))|x\in X\}$. We denote by I the unit interval $[0,1]$ and by Q the Hilbert cube. A proper map $p:E\to B$ between locally compact, separable metric ANR's is an *approximate fibration* [2] if given an open cover $\mathcal{U}$ of B, a space X and maps $h:X\to E$, $H:X\times I\to B$ such that $ph=H_0$, then there is a homotopy $\tilde{H}:X\times I\to E$ such that $\tilde{H}_0=h$ and $p\tilde{H}$ and H are $\mathcal{U}$-close, where $H_t(x)=H(x,t)$. Let $\underline{E}=(E_i,q_{ij})$ and $\underline{B}=(B_i,r_{ij})$ be inverse sequences of compacta and let $\underline{p}=(p_i)$ be a sequence of maps $p_i:E_i\to B_i$. $\underline{p}:\underline{E}\to\underline{B}$ is a *level map* if for any i and $j\geqq i$, $p_iq_{ij}=r_{ij}p_j$. A map $p:E\to B$ between compacta is a *shape fibration* [11] if there is a level map $\underline{p}:\underline{E}\to\underline{B}$ of compact ANR-sequences with invlim $\underline{E}=E$, invlim $\underline{B}=B$ and invlim $\underline{p}=p$ satisfying the following property; for each i and

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