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SHAPE FIBRATIONS AND FIBER SHAPE EQUIVALENCES, I

By

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0. Introduction.

In [6], Coram and Duvall introduced approximate fibrations and Mardešić and Rushing [11] generalized this and defined shape fibrations. For compact ANR's, shape fibrations agree with approximate fibrations. M. Jani, analogous to fiber maps, defined fiber morphisms and fiber shape equivalences [8]. In [4], Chapman proved the Complement Theorem, i.e., if X and Y are Z-sets in the Hilbert cube Q, then X and Y have the same shape (i.e., $\mathrm{Sh}(X)=\mathrm{Sh}(Y)$, see [2]) iff $Q-X$ and $Q-Y$ are homeomorphic.

In this paper, we define notions of fiber fundamental sequences and fiber shape equivalences and prove that if a fiber fundamental sequences between approximate fibrations is a shape equivalence, then it is a fiber shape equivalence. Also, we prove the following: Let E, E' and B be compacta in the Hilbert cube Q and let $E, E'\subset Q$ be Z-sets. Then a map $p: E\rightarrow B$ over B is fiber shape equivalent to a map $p': E'\rightarrow B$ over B if and only if there is a homeomorphism $h: Q-E\cong Q-E'$ such that for each $b\in B$ and each neighborhood W' of $p'^{-1}(b)$ in Q, there is a neighborhood W of $p^{-1}(b)$ in Q such that $h(W-E)\subset W'-E'$.

All spaces considered will be metrizable. If x and y are points of a metric space, $d(x, y)$ denotes the distance from x to y. A proper map $p: E\rightarrow B$ between locally compact, separable metric ANR's is an *approximate fibration* [6] if given an open cover $\mathcal{U}$ of B, a space X and maps $h: X\rightarrow E$, $H: X\times I\rightarrow B$ such that $ph=H_0$, then there is a homotopy $\tilde{H}: X\times I\rightarrow E$ such that $\tilde{H}_0=h$ and H and $p\tilde{H}$ are $\mathcal{U}$-close, where $H_t(x)=H(x, t)$. Let $\underline{E}=(E_i, q_{ij})$ and $\underline{B}=(B_i, r_{ij})$ be inverse sequences of compacta and let $\underline{p}=(p_i)$ be a sequence of maps $p_i: E_i\rightarrow B_i$. Then $\underline{p}: \underline{E}\rightarrow\underline{B}$ is a *level map* if for any i and $j\geqq i$, $p_i q_{ij}=r_{ij}p_j$. A map $p: E\rightarrow B$ between compacta is a *shape fibration* [11] if there is a level map $\underline{p}: \underline{E}\rightarrow\underline{B}$ of compact ANR-sequences with $\varprojlim\underline{E}=E$, $\varprojlim\underline{B}=B$ and $\varprojlim\underline{p}=p$ satisfying the following property; for each i and $\varepsilon>0$ there is $j\geqq i$ and $\delta>0$ such that for any space X and any $h: X\rightarrow E_j$, $H: X\times I\rightarrow B_j$ with $d(p_j h, H_0)=\sup\{d(p_j h(x), H_0(x))|x\in X\}<\delta$, there is a homotopy

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