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COMPLETIONS AND CO-PRODUCTS OF HEYTING ALGEBRAS

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A Heyting algebra is not only a lattice theoretic object, but is also related to the intuitionitic logic and a topological space and others. In this paper, we shall investigate about completions and co-products of Heyting algebras.

In §1, we shall study about a Stone space as a complete Heyting algebra, more precisely as a completion of some distributive lattice. In §2, the canonical completion of a Heyting algebra will be studied. Some proofs in §1, §2 and §6 are done intuitionistically. Those cares are necessary for §6. A co-product of Heyting algebras is defined in §3. In §4, we shall study the space of maximal ideals and Wallman-compactifications and Stone-Cech-compactifications in the Heyting algebraic view. The relationships between some properties, completions and co-products defined in the previous sections will be discussed in §5. Complete Heyting algebras in a Heyting extention will be studied in §6.

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§1. An open algebra of a Stone space

We shall use usual lattice-theoretic notations and set-theoretic ones.

DEFINITION 1.1. A lattice L is distributive, if $a\wedge(b\vee c)=(a\wedge b)\vee(a\wedge c)$ holds.

A lattice L is bounded, if it has the least element $\mathbb{0}$ and the greatest element $\mathbb{1}$.

A lattice L is a bounded distributive lattice, if it is bounded and distributive.

A lattice L is a Heyting algebra, if it is a bounded distributive lattice and relatively pseudo-compemented. We denote the relative pseudo-complement by $a\Rightarrow b$, where $x\leq a\Rightarrow b$ if and only if $a\wedge x\leq b$.

DEFINITION 1.2. A lattice L is complete if the least upper bound for any

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