TSUKUBA J. MATH
Vol. 5 No 1 (1981). 153–163

A PRERADICAL WHICH SATISFIES THE PROPERTY THAT EVERY WEAKLY DIVISIBLE MODULE IS DIVISIBLE

By

Yasuhiko TAKEHANA

Recently M. Sato has studied a radical satisfying the property that every weakly codivisible module is codivisible in [6]. In this paper we study preradicals for which every weakly divisible module is divisible. We characterize an idempotent preradical with this property in Theorems 1.7 and 1.8. Moreover we characterize an idempotent preradical for which every weakly divisible module is injective in Proposition 1.10. Dually we consider a radical for which every weakly codivisible module is projective in Proposition 1.13.

In § 2 we study a preradical t which has the property that $t(E/K)=(t(E)+K)/K$ holds for any injective module E and any submodule K of E. We call this an injectively epi-preserving preradical and characterize in Theorem 2.1.

Dualizing this, we study a preradical which has the property that $t(K)=K\cap t(P)$ holds for any projective module P and any submodule K of P, and we have Theorem 2.4.

Last we give examples of these preradicals.

1. Weakly (co-) divisible modules and (co-) divisible modules.

Throughout this paper R is a ring with a unit element, every right R-module is unital and Mod-R is the category of right R-modules. A subfunctor of the identity functor of Mod-R is called a preradical. A preradical t is called idempotent (resp. radical) if $t(t(M))=t(M)$ (resp. $t(M/t(M))=0$) for any module M. For a preradical t we put $\mathcal{T}_t=\{M\in\text{Mod-}R\,;\,t(M)=M\}$ and $\mathcal{F}_t=\{M\in\text{Mod-}R\,;\,t(M)=0\}$ whose elements are said to be torsion and torsionfree modules respectively. We say that M is divisible (resp. weakly divisible) if $\text{Hom}_R(\_, M)$ preserves the exactness for every exact sequence $0\to A\to B\to C\to 0$ with $C\in\mathcal{T}_t$ (resp. $B\in\mathcal{T}_t$). Dually we say that M is codivisible (resp. weakly codivisible) if $\text{Hom}_R(M, \_)$ preserves the exactness for every exact sequence $0\to A\to B\to C\to 0$ with $A\in\mathcal{F}_t$ (resp. $B\in\mathcal{F}_t$).

To begin with we study a fundamental property of weakly divisible modules.

Received December 2, 1980. Revised February 16, 1981.