TSUKUBA J. MATH.
Vol 5 No. 1 (1981). 67–83

A STUDY OF RINGS WITH TRIVIAL PRERADICAL IDEALS

(Dedicated to Professor Goro Azumaya for the celebration of his sixtieth birthday)

By

Hisao KATAYAMA

0. Introduction.

Our purpose is to study those rings without non-trivial preradical ideals of idempotent preradicals (or exact radicals), supplying the cases of idempotent radicals by [2], of left exact preradicals by [1, 6, 14, 17] and of left exact radicals by [2, 6]. In Theorem 3.1, we shall show that a ring R has no non-trivial idempotent preradical ideals if and only if every nonzero left ideal is a generator for R-mod (left G-ring). Generalizing this, we consider those rings whose nonzero finitely generated (or cyclic, essential) left ideals are generators. We shall give several examples which distinguish those rings to be refered.

1. Preliminaries.

This section consists of a list of definitions and properties of some type of preradicals treated in this paper. In particular, we shall give the bijections of those preradicals for Morita equivalent rings.

Let R be a ring with identity and R-mod the category of all unital left R-modules. A functor $\sigma: R\text{-mod}\rightarrow R\text{-mod}$ is called a *preradical* if $\sigma(M)$ is a submodule of M for each $M\in R$-mod and $\sigma(M)\alpha\subseteq\sigma(N)$ for each morphism $\alpha: M\rightarrow N$ in R-mod. A preradical σ is called an *idempotent preradical* (resp. a *radical*) if $\sigma(\sigma(M))=\sigma(M)$ (resp. $\sigma(M/\sigma(M))=0$) for all $M\in R$-mod. A preradical is called *left exact* (resp. *cohereditary*) if it is kernel preserving (resp. epi-preserving). Every left exact (resp. cohereditary) preradical is idempotent (resp. a radical). A preradical is called a *cotorsion radical* (resp. an *exact radical*) if it is an idempotent cohereditary radical (resp. a left exact cohereditary radical). For preradicals σ_1 and σ_2, $\sigma_1\leq\sigma_2$ means that $\sigma_1(M)\subseteq\sigma_2(M)$ for all $M\in R$-mod.

To a preradical σ for R-mod, we associate the pair $(\mathcal{T}_\sigma, \mathcal{F}_\sigma)$ of classes of

Received September 8, 1980.