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ON ORTHOGONALITY IN S. I. P. SPACES

By

K. R. UNNI and C. PUTTAMADAIAH

Abstract: In this paper we study the orthogonality in an s. i. p. space and prove a theorem of Giles under weaker conditions.

Let X be a complex (real) vector space. A complex (real) function $[,]$ on $X\times X$ is called a semi-inner-product (s. i. p.) on X if it satisfies the following properties:

(i) $[x+y, z]=[x, z]+[y, z]$

(ii) $[\lambda x, y]=\lambda[x, y]$

(iii) $[x, x]>0$ for $x\neq 0$

(iv) $|[x, y]|^{2}\leqq[x, x]\cdot[y, y]$

for all x, y, z in X and for all scalars λ. A vector space X with an s. i. p. is called a semi-inner-product space (s. i. p. space).

In what follows we shall consider complex s. i. p. spaces. An s. i. p. space X is normed linear space with the norm given by $\|x\|=[x, x]^{1/2}$ for x in X ([4], Theorem 2, p. 31) and the topology on an s. i. p. space is the one induced by this norm. It is also known ([4], p. 31) that every normed linear space can be made into an s. i. p. space (in general in infinitely many different ways). An s. i. p. space is said to have the homogeneity property when the s. i. p. satisfies

(v) $[x, \lambda y]=\bar{\lambda}[x, y]$ for all x, y in X and for all complex numbers λ. Giles [2] showed that every normed linear space can be represented as an s. i. p. space with the homogeneity property.

An s. i. p. space X is said to be continuous if for every x, y in X

$$Re[y, x+\lambda y]\rightarrow Re[y, x] \quad \text{for all real } \lambda\rightarrow 0.$$

We say that in an s. i. p. space X, x is orthogonal to y if $[y, x]=0$.

If M is a subset of an s. i. p. space X, let

$$M^{\perp}=\{x\in X:[m, x]=0 \quad \text{for all } m\in M\}.$$

In this paper we study the properties of $M^{\perp}$; in particular we show that $M^{\perp}$ is always closed and establish a decomposition theorem for s. i. p. spaces.

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