

RELATIVE FREE ENERGY AND ITS APPLICATION TO SPEED CHANGE MODEL

By

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§ 1. Introduction.

The purpose of this paper is to study a continuous spin system by introducing the relative free energy* which plays a role of the free energy in discrete spin systems.

Although the usual free energy is a very important functional, it can be available to only identically distributed 2-spin lattice systems. About such systems, refer to Holley [1], [2]. On the other hand, our relative free energy enables us to deal with spin systems with state space not necessarily discrete nor identically distributed. In Section 3, we consider a 1-dimensional lattice spin system with continuous state space to show that the relative free energy does not increase with time and especially it strictly decreases if the initial state is not Gibbsian; this is one of the main results of this paper. From this fact, moreover we conclude that the time evolution of any shift invariant non-Gibbsian state converges to an equilibrium state. The precise definitions of the relative free energy et al are given in Section 2.

§ 2. Definition of relative free energy.

Let Ω_* be a compact Hausdorff space with Second countability axiom, and let \mathcal{B}_* be its topological Borel field. We suppose that a probability measure ν_* is given on $(\Omega_*, \mathcal{B}_*)$. We denote the two sided countable direct product of copies of $(\Omega_*, \mathcal{B}_*, \nu_*)$ by $(\Omega, \mathcal{B}, \nu)$. Let C_n be the family of all $(-n, n)$ -cylinder sets i.e. the sets of the form $\{\omega \in \Omega; \omega_i \in E_i, i=0, \pm 1, \dots, \pm n\}$, $E_i \in \mathcal{B}_*$, and we denote by \mathcal{B}_n the σ -algebra generated by C_n .

The restriction to \mathcal{B}_n of a probability measure μ on \mathcal{B} is denoted by $\mu^{(n)}$. Moreover the set of all probability measures on $\mathcal{B}(\mathcal{B}_n)$ is denoted by $\mathcal{P}(\mathcal{P}_n)$.

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* In [5], Sullivan defined the relative free energy which is similar to ours and obtained the similar results. But his free energy depends on the special kind of invariant measure of the generator. Therefore it is hardly possible to deal with our model.