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ON SYMMETRY OF KNOTS

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§ 1. Introduction

A knot K in a 3-sphere S^3 is said to have period n [9] [13] (or to be a *periodic* knot of order n) if there is a rotation of S^3 with period n and axis A, where $A \cap K = \phi$, which leaves K invariant. (This definition is now equivalent to the original definition due to the positive solution to Smith Conjecture.)

One of the main problems is to determine periods of a given knot and so far, several necessary conditions for K to have period n have been found. See [1], [5], [7], [9], [13].

In this paper, we prove a few additional conditions using the covering linkage invariants which will be explained below.

Let J_n be a set of n letters, 1, 2, $\cdots$, n, and $\mathcal{S}(J_n)$ the groups of all permutations on J_n. Thus $\mathcal{S}(J_n)$ is isomorphic to the symmetric group of order n!

Let Γ be a finite transitive permutation group, i.e., Γ is a transitive subgroup of $\mathcal{S}(J_n)$.

An epimorphism $\theta: G \to \Gamma \leqq \mathcal{S}(J_n)$ is called, in this paper, a representation of G of degree n.

Two representations $\theta_1, \theta_2: G \to \Gamma$ will be called *equivalent* [4], is symbols $\theta_1 \equiv \theta_2$, if there is an inner automorphism $\rho: \mathcal{S}(J_n) \to \mathcal{S}(J_n)$ such that $\rho\theta_1 = \theta_2$.

Let M be a 3-manifold and $G = \pi_1(M)$. To each representation of G of degress n, there is defined uniquely (up to homeomorphism) an n-sheeted covering space $\tilde{M}$ of M. Equivalent representations define homeomorphic covering spaces.

Let K be a knot in S^3 and let $M = S^3 - K$. In this paper, $\pi_1(S^3 - K)$ is denoted by $G(K)$. A representation $\theta: G(K) \to \Gamma \leqq \mathcal{S}(J_n)$ defines the covering space $\tilde{M}$ of M, called the *unbranched* covering space of K in S^3. It is known that $\tilde{M}$ is of the form $M^* - \tilde{K}$ for some orientable closed 3-manifold M^* and a knot (or link) $\tilde{K}$ in M^*. The "completion" M^* of $\tilde{M}$ is called the *branched covering space* of

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