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STRUCTURE OF COMPLEX TORI WITH THE AUTOMORPHISMS OF MAXIMAL DEGREE

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§ 0. Introduction.

There is a classical result that an elliptic curve with a non-trivial automorphism is determined uniquely by the automorphism. So that we expect also in higher dimensional cases many automorphisms will affect the structure of the base space. The formulation of this idea is as follows: Let T be an n-dimensional complex torus with an automorphism φ and $A\in GL_n(\boldsymbol{C})$ be the complex representation of φ. Then $A\Omega=\Omega N$ for a period matrix Ω of T and $N\in GL_{2n}(\boldsymbol{Z})$. Hence we have

$$\begin{pmatrix} A & 0 \\ 0 & \bar{A} \end{pmatrix}\begin{pmatrix} \Omega \\ \bar{\Omega} \end{pmatrix}=\begin{pmatrix} \Omega \\ \bar{\Omega} \end{pmatrix}N,$$

where $-$ denotes the complex conjugate. Thus all the eigenvalues of A are units of algebraic number fields of degree$\leqq 2n$. We shall study here the structure of T when one of the eigenvalues has the degree $2n$. In this case all of them have the degree $2n$ and let us call φ (or A) an *automorphism of degree* $2n$. The main results are stated in the following section. The author would like to express his hearty thanks to Prof. K. Uchida and Prof. T. Takeuchi for valuable suggestions.

§ 1. Statement of main results.

In the sequel we assume that φ has the degree $2n$ except Lemma 2.1 and Theorem 2.2.

First we fix our notation as follows: We consider only the case $n\geqq 2$, $\{\alpha_1, \cdots, \alpha_n\}$: the eigenvalues of A,

$$K=\boldsymbol{Q}(\alpha_1, \cdots, \alpha_n, \bar{\alpha}_1, \cdots, \bar{\alpha}_n),$$

$$G=Gal(K/\boldsymbol{Q}),$$

ρ : the complex conjugate mapping,
φ_i : the isomorphism $\boldsymbol{Q}(\alpha_1)\to\boldsymbol{Q}(\alpha_i)$ over $\boldsymbol{Q}$, where $\varphi_i(\alpha_1)=\alpha_i$ $(i=1, \cdots, n)$,

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