A LIMIT THEOREM FOR CONDITIONAL RANDOM WALK

By

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§ 1. Introduction

Let $X, X_1, X_2, \ldots$, be a sequence of real valued independent, identically distributed random variables defined on a probability space $(\Omega, \beta, P)$ such that

(I) $EX=0$ and $0<\sigma^2 := E(X^2) < \infty$.

Let $\{S_n; n \geq 0\}$ be a random walk defined by

$S_0 = 0$ and $S_n := X_1 + \cdots + X_n$ for $n \geq 1$.

The main purpose of this paper is to show the following

Theorem. In addition to (I) suppose that

(II) $E(X^a \log(1+|X|)^a) > \infty$

for some constant $\alpha > 1$. Then

$P(S_n/a\sqrt{n} \leq x | S_k > 0 \text{ for every } k, 1 \leq k \leq n) \overset{d}{\rightarrow} 1 - \exp(-x^2/2)$

(convergence in distribution of distribution functions on the semi-infinite interval $[0, \infty)$).

The result of Theorem (without the condition (II)) was announced by Spitzer [19], page 162, in a footnote “Added in proof”. But the proof was not published. The rigorous proof was given by Iglehart [10], Proposition 2.1 under the condition that random walk has finite third absolute moment and in addition the maximal span one when it is integer-valued. It has been open whether his condition is necessary or not (see Iglehart [11], page 177). Our Theorem asserts that his condition is not necessary.

In § 2 we discuss asymptotic property of probability distributions of random walk. In § 3 we prepare several lemmas which play important role in our proof of Theorem. In § 4 we prove our Theorem.

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