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A LIMIT THEOREM FOR CONDITIONAL RANDOM WALK

By

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§ 1. Introduction

Let $X, X_1, X_2, \cdots,$ be a sequence of real valued independent, identically distributed random variables defined on a probability space (Ω, β, P) such that

(I) $$EX=0 \quad \text{and} \quad 0<\sigma^2:=E(X^2)<\infty.$$

Let $\{S_n; n\geqq 0\}$ be a random walk defined by

$$S_0=0 \quad \text{and} \quad S_n:=X_1+\cdots+X_n \text{ for } n\geqq 1.$$

The main purpose of this paper is to show the following

THEOREM. *In addition to* (I) *suppose that*

(II) $$E(X^2[\log(1+|X|)]^{\alpha})>\infty$$

for some constant $\alpha>1$. *Then*

$$P(S_n/\sigma\sqrt{n}\leqq x|S_k>0 \textit{ for every } k, 1\leqq k\leqq n) \xrightarrow{d} 1-\exp(-x^2/2)$$

(*convergence in distribution of distribution functions on the semi-infinite interval* $[0, \infty)$).

The result of Theorem (without the condition (II)) was announced by Spitzer [19], page 162, in a footnote " Added in proof ". But the proof was not published. The rigorous proof was given by Iglehart [10], Proposition 2.1 under the condition that random walk has finite third absolute moment and in addition the maximal span one when it is integer-valued. It has been open whether his condition is necessary or not (see Iglehart [11], page 177). Our Theorem asserts that his condition is not necessary.

In § 2 we discuss asymptotic property of probability distributions of random walk. In § 3 we prepare several lemmas which play important role in our proof of Theorem. In § 4 we prove our Theorem.

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