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CONSTRUCTIONS OF MODULAR FORMS BY MEANS OF TRANSFORMATION FORMULAS FOR THETA SERIES

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Let F be a positive integral symmetric matrix of degree m, and Z a variable on the Siegel space H_n of degree n. Let Φ be a spherical function of order ν with respect to F which is of the form

$$\Phi(G)=\begin{cases} 1 & (\nu=0) \\ |{}^tGF^{1/2}\eta|^{\nu} & (\nu>0) \end{cases} \qquad \text{for } m\times n \text{ complex matrices } G$$

with an $m\times n$ matrix η such that ${}^t\eta\eta=0$ if $\nu>1$.

We define a theta series associated with F by setting

$$\theta_{F,U,V}(Z;\Phi)=\sum_{G}\Phi(G+V)\exp(\mathrm{tr}(Z^t(G+V)F(G+V)+2^t(G+V)U)),$$

where U, V are $m\times n$ real matrices, tr denotes the trace of a corresponding square matrix and G runs through all $m\times n$ integral matrices. We write simply $\theta_{F,U,V}(Z)$ for the theta series $\theta_{F,U,V}(Z;\Phi)$ when Φ is of order 0.

For congruence subgroups of $SL_2(\boldsymbol{Z})$ the transformation formulas for theta series of degree 1 associated with F are well known. For example, we can find transformation formulas for theta series of degree 1 in [7], [8], in which multipliers are explicitly determined. Transformation formulas for the theta series $\theta_{F,U,V}(Z;\Phi)$ of degree $n\geq 1$ are also established in [1] in the case where F is even and U, V are zero (the condition on U, V is not necessary if Φ is of order 0 [9]). Using these results we can get many examples of Siegel modular forms for congruence subgroups.

In this paper we determine a transformation formula for the theta series $\theta_{F,U,V}(Z;\Phi)$ associated with a positive integral symmetric matrix F and any real matrices U, V and using this, we get some examples of cusp forms for some congruence subgroups Γ' of $Sp_n(\boldsymbol{Z})$. Cusp forms of weight $n+1$ for Γ' induce differential forms of the first kind on the nonsingular model of the modular function field with respect to Γ'. Our result shows that the geometric genus of the nonsingular model of the modular function field with respect to Γ' is positive.

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