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ON A PROBLEM ABOUT SKOLEM'S PARADOX OF TAKEUTI'S VERSION

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§1. Introduction.

In his paper [2], Takeuti gave a theorem as a proof-theoretical interpretation of so-called Skolem's paradox. He showed as a corollary that Gödel-Bernays set theory with the additional axioms

$$\forall X \exists x \in \omega (f(x)=X)$$

is consistent, where f is a newly introduced function symbol.

Concerning it, we are interested here in the question whether we can add consistently the axiom schema

$$\vec{\forall}(\mathfrak{F}(0) \wedge \forall x(\mathfrak{F}(x) \rightarrow \mathfrak{F}(x+1)) \rightarrow \forall x \in \mathfrak{F}(x))$$ [1),2)]

bisides with an axiom $\forall x \exists y \in \omega (f(y)=x)$ to any consistent set theory.

We show in this paper that the above question is affirmative for any con sistent extension of ZF.[3)]

§2. Results.

2.1. CONVENTION. In the first order predicate calculus with equality, we regard the equality axioms as logical axioms. It means that equility axioms for any language are provided always tacitly.

f, g, h, ... denote function symbols.

$\mathfrak{s}$, $\mathfrak{t}$, $\mathfrak{u}$, ... denote terms.

$\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, ... denote formulae.

$\boldsymbol{s}$ is a unary function symbol and $\boldsymbol{s}(\mathfrak{t})$ is abbreviated by $\mathfrak{t}'$.

seq is a binary function symbol and **seq**$(\mathfrak{s}, \mathfrak{t})$ is abbreviated by $\mathfrak{s}^{\prime}\mathfrak{t}$.

P, Q, R, ... denote predicate symbols.

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1) $\vec{\forall}\mathfrak{A}$ denotes the universal closure of a formula $\mathfrak{A}$.

2) The formula $\mathfrak{F}(0)$ contains possibly the newly introduced function symbol f.

3) See [1], where the rather folklore result that the question is affirmative for any consistent extension of ZFC is shown.

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