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ON THE EQUATIONS DEFINING A PROJECTIVE CURVE EMBEDDED BY A NON-SPECIAL DIVISOR

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Introduction. Let C be a complete reduced irreducible curve of arithmetic genus g over an algebraically closed field K. Let L be a very ample invertible sheaf of degree d on C, and let $\phi_L: C \subsetneq \boldsymbol{P}^{h^0(L)-1}$ be the projective embedding by means of a basis of $\Gamma(L)$. Then the following results are known:

(A) Assume that C is smooth over K.

(0) (D. Mumford [5]) L is normally generated, if $d \geq 2g+1$.

(1) (B. Saint-Donat [7]) The largest homogeneous ideal I defining $\phi_L(C)$, i.e., $I=\operatorname{Ker}[S\Gamma(L) \rightarrow \bigoplus_{m\geq 0} \Gamma(L^m)]$, is generated by its elements of deree 2, if $d \geq 2g+2$.

(2) (B. Saint-Donat [7]) I is generated by its elements of degree 2 and 3, if $d \geq 2g+1$.

(B) (T. Fujita [1]) The statements (0) and (1) in (A) are true without the assumption that C is smooth over K.

The purposes of the present paper are that we improve the second result (2) of Saint-Donat and that we construct some related examples (corollary 1.4, Example 2.4 and Proposition 3.1).

Notation and Terminology. We fix an algebraically closed field K of characteristic $p \geqq 0$ throughout the paper. We use the word "variety" to mean a reduced irreducible scheme of finite type and proper over K, and "curve" to mean a variety of dimention 1.

For a finite dimensional vector space V over K, $S^m V$ means the m-th symmetric power of V and SV means the symmetric algebra of V, i.e., $SV = \bigoplus_{m\geqq 0} S^m V$.

Let L be an invertible sheaf on a projective variety X. We denote by L^m the m-th tensor product $L^{\otimes m}$. For the vector space of global sections $\Gamma(L)$, we define I and I_m ($m \geqq 1$), by

$$I = I(L) = \operatorname{Ker}[S\Gamma(L) \rightarrow \bigoplus_{m\geqq 0} \Gamma(L^m)],$$

and

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