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CONVERGENCE OF MOMENTS IN THE CENTRAL LIMIT THEOREM FOR STATIONARY ϕ-MIXING SEQUENCES

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1. Introduction and result.

Let $\{X_j, \ -\infty<j<\infty\}$ be a strictly stationary sequence of random variables centered at expectations with finite variance, which satisfies ϕ-mixing condition

$$\sup |P(A\cap B)-P(A)P(B)|/P(A)\leqq\phi(n)\downarrow 0 \quad (n\rightarrow\infty). \tag{1.1}$$

Here the supremum is taken over all $A\in\mathcal{M}_{-\infty}^{k}$ and $B\in\mathcal{M}_{k+n}^{\infty}$, and $\mathcal{M}_{a}^{b}$ denotes the σ-field generated by X_j $(a\leqq j\leqq b)$. Let $S_n=X_1+\cdots+X_n$ and $\sigma_n{}^2=ES_n{}^2$, $n=1, 2, \cdots$.

For independent random variables, Brown [1 and 2] has shown that the Lindeberg condition of order $\nu\geqq 2$ is necessary and sufficient for the central limit theorem and the convergence of $E|S_n/\sigma_n|^{\nu}$ towards the corresponding moment of the normal distribution. For dependent random variables, such a result seems less well-known. We study here the convergence of moments for stationary ϕ-mixing sequences.

THEOREM. *Let* $\{X_j\}$ *satisfy* (1.1). *If* $EX_1{}^{2m}<\infty$ *for some integer* $m\geqq 2$, *and if*

$$\sigma_n{}^2=\sigma^2 n(1+o(1)) \tag{1.2}$$

as $n\rightarrow\infty(\sigma>0)$, *then*

$$E(S_n/\sigma_n)^{2m}\rightarrow\beta_{2m} \quad (n\rightarrow\infty), \tag{1.3}$$

where β_ν *is the* ν*th absolute moment of* $N(0, 1)$.

We remark that under the assumptions of the theorem X_j satisfies the central limit theorem (cf. [4, Theorem 18.5.1]). Also remark that any other conditions beyond (1.1) on the decays of mixing coefficients $\phi(n)$ are not required.

2. Preparatory lemmas.

LEMMA 1 [4, Theorem 17.2.3]. *Suppose that* (1.1) *is satisfied and that* ξ *and* η *are measurable with respect to* $\mathcal{M}_{-\infty}^{k}$ *and* $\mathcal{M}_{k+n}^{\infty}$ $(n\geqq 0)$ *respectively. If* $E|\xi|^p<\infty$ *and* $E|\eta|^q<\infty$ *for* $p, q>1$ *with* $(1/p)+(1/q)=1$, *then*

$$|E(\xi\eta)-E(\xi)E(\eta)|\leqq 2\{\phi(n)E|\xi|^p\}^{1/p}\{E|\eta|^q\}^{1/q}. \tag{2.1}$$

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