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PHASE TRANSITIONS IN CLASSICAL HEISENBERG MODELS

By
Koji KURODA

§ 1. Introduction and notations

In this paper we extend the result of Malyshev [2] and show the existence of the phase transitions in Heisenberg model which has more than three phases by using the method of Peierls. Also we discuss the phase transitions in finite spin systems.

In the corollary of therem 1 in § 2 we also show that phase transitions occur in classical Heisenberg antiferromagnet under sufficiently low temperature.

As to the finite spin systems the existence of phase transition can be proved in a similar way to the previous one.

Now we give the definition of Gibbsian random field of the system and introduce some notations used below. We take a compact seperable metric space S as a spin space and denote by μ a non negative finite measure on the topological Borel field in S.

We consider a spin system on a 2-dimensional lattice T with a nearest neighbour potential $U(s_1, s_2)$, where $U(s_1, s_2)$ is a real function on $S\times S$ which is measurable and bounded from below.

For a finite subset $V=\{t_1, \cdots, t_v\}\subset T$, we associate a σ-field $\mathscr{B}_V$ generated by $\{\omega(t); t\in V\}$ $(\omega\in\Omega=S^T)$ and let $\mathscr{B}=\mathscr{B}_T$. A probability measure P on $(\Omega, \mathscr{B})$ is called a Gibbsian random field if for each finite set V and for each atom

$$A=\{\omega\in\Omega;\ \omega(t_i)=s_i\ (i=1, \cdots, v)\}\ \text{of}\ \mathscr{B}_V$$

$$P(A|\mathscr{B}_{V^c})=q_{V,\omega}(A)\quad a.e.\ P \tag{1.1}$$

where

$$q_{V,\omega}(A)=\Xi(V, \omega)^{-1}\exp\{-\beta U_V(s_1, \cdots s_v|\omega(t))\}$$

$$U_V(s_1, \cdots, s_v|\omega(t))=\frac{1}{2}\sum_{|t_i-t_j|=1}U(s_i, s_j)+\sum_{i=1}^{V}\sum_{\substack{t\in V^c\\|t-t_i|=1}}U(s_i, \omega(t))$$

and

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