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EQUIVARIANT POINT THEOREMS

(Dedicated to Professor A. Komatu on his 70th birthday)

By

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1. Introduction.

This paper is a continuation of my previous paper [13], and is concerned with generalizations of the following two classical theorems on a continuous map f of an n-sphere S^{n} to itself.

THEOREM 1.1. *If the degree of* f *is even then there exists* $x\in S^{n}$ *such that* $f(-x)=f(x)$.

THEOREM 1.2. *If the degree of* f *is odd then there exists* $x\in S^{n}$ *such that* $f(-x)=-f(x)$.

Throughout this paper, a prime p is fixed, and $G=\{1, T, \cdots, T^{p-1}\}$ will denote a cyclic group of order p.

Generalizing the situation in the above theorems, we shall consider the following problems.

PROBLEM 1. *Let* $f: N\rightarrow M$ *be a continuous map between G-spaces. Under what conditions does* f *have an equivariant point, i.e., a point* $x\in N$ *such that*

$$f(T^{i}x)=T^{i}f(x) \tag{1.1}$$

for $i=1,2, \cdots, p-1$?

PROBLEM 2. *Let* $f: L\rightarrow M$ *and* $g: L\rightarrow N$ *be continuous maps of a space* L *to G-spaces* M *and* N. *Under what conditions do there exist* p *points* $x_{1}, \cdots, x_{p}\in L$ *such that*

$$f(x_{i+1})=T^{i}f(x_{1}), \ g(x_{i+1})=T^{i}g(x_{1}) \tag{1.2}$$

for $i=1,2, \cdots, p-1$?

We shall denote by $A(f)$ the set of points $x\in N$ satisfying (1.1), and by $A(f, g)$ the set of points $(x_{1}, \cdots, x_{p})\in L^{p}$ satisfying (1.2).

If $L=N$ in Problem 2, then $A(f, \mathrm{id})$ may be identified with $A(f)$. Therefore

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