TSUKUBA J. MATH.
Vol. 2 (1977). 35—68

ON CONJUGATE LOCI AND CUT LOCI OF COMPACT SYMMETRIC SPACES I

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Introduction

Let (M, g) be a compact connected Riemannian manifold. Fix a point o of M and denote by $T_o(M)$ the tangent space of M at o. Let Exp: $T_o(M) \longrightarrow M$ be the exponential map of (M, g) at o. A tangent vector $X \in T_o(M)$ is called a *tangential conjugate point* of (M, g), if Exp is degenerate at X. The set $\tilde{Q}$ of all tangential conjugate points of (M, g) in $T_o(M)$ is called the *tangenital conjugate locus* of (M, g) in $T_o(M)$. The image $Q = \mathrm{Exp}\,\tilde{Q}$ of $\tilde{Q}$ under Exp is called the *conjugate locus* of (M, g) with respect to o.

Let γ: $[0, \infty) \longrightarrow M$ be a geodesic of (M, g) (parametrized by arc-length) emanating from o. Let $X_1 = \dot{\gamma}(0) \in T_o(M)$ denote the initial tangent vector of γ. Assume that the set of $t \in [0, \infty)$ such that $tX_1 \in \tilde{Q}$ is not empty and let t_0 be the infimum of this set. Then the tangent vector $t_0 X_1$ is called the *tangential first conjugate point along* γ. The set $\tilde{F}$ of all $X \in T_o(M)$ which is the tangenital first conjugate point along some geodesic γ emanating from o, is called the *tangenital first conjugate locus* of (M, g) in $T_o(M)$. The image $F = \mathrm{Exp}\,\tilde{F}$ of $\tilde{F}$ under Exp is called the *first conjugate locus* of (M, g) with respect to o.

Let again γ: $[0, \infty) \longrightarrow M$ be a geodesic emanating from o and $X_1 = \dot{\gamma}(0)$. Let $\bar{t}_0$ be the supremum of the set of $t \in [0, \infty)$ such that $\gamma|[0, t]$ is a minimal geodesic segment from o to $\gamma(t)$. The number $\bar{t}_0$ is always finite since M is compact. Then the tangent vector $\bar{t}_0 X_1$ is called the *tangenital cut point along* γ. The set $\tilde{C}$ af all $X \in T_o(M)$ which is the tangenital cut point along some geodesic γ emanating from o, is called the *tangenital cut locus* of (M, g) in $T_o(M)$. The image $C = \mathrm{Exp}\,\tilde{C}$ of $\tilde{C}$ under Exp is called the *cut locus* of (M, g) with respect to o.

In the present article, we shall study the structures of the conjugate locus, the first conjugate locus and the cut locus of a compact symmetric space.

Helgason [3] showed by a group theoretical method that the conjugate locus of a compact connected Lie group M, endowed with a bi-invariant Riemannian metric g, is nicely stratified in the sense that it is the disjoint union of smooth submani-

Received December 28, 1977