TSUKUBA J. MATH.
Vol. 1 (1977), 157–162

A CHARACTERIZATION OF COMPLEX PROJECTIVE SPACES BY LINEAR SUBSPACE SECTIONS

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1. Introduction

It is conjectured in [2] that a complex projective space will be characterized from the standpoint of the positivity of sectional curvature. This conjecture is partially supported. Namely, a compact Kähler manifold (M, g) with positive curvature is biholomorphically homeomorphic to a complex projective space, if, for examples, one of the following conditions is satisfied;

i) $\dim_{C}M=2$ ([2]),

ii) the Kähler metric g is Einstein ([1]),

iii) the group of holomorphic transformations acts on M transitively ([6]) and

iv) $\dim_{C}M=3$ or 4 and $H^{*}(M;\ \boldsymbol{Z})\cong H^{*}(\mathbf{P}_{n}(\mathbf{C});\ \boldsymbol{Z})$, $n=\dim_{C}M$ ([4]).

These conditions play essential role in each result.

In this connection, we are in a position to consider the following assertion.

ASSERTION If a compact complex manifold M admits a closed complex submanifold, in particular, a closed complex hypersurface which is biholomorphically homeomorphic to a complex projective space, then M itself is biholomorphically homeomorphic to a complex projective space.

If this assertion is verified, the conjecture due to Frankel can be reduced to the following conjecture.

CONJECTURE A compact Kähler manifold with positive curvature will admit a closed complex submanifold endowed with a Kähler metric of positive curvature.

Of course, the submanifold of positive curvature may not be a Kähler submanifold of the ambient manifold.

In general, the assertion is false. For example, a product manifold $\mathbf{P}_{n}(\mathbf{C})\times M$, where M is a compact complex manifold, has $\mathbf{P}_{n}(\mathbf{C})$ as a closed complex submanifold, but the total manifold can never be biholomorphically homeomorphic to a complex projective space. Hence, the submanifold in the assertion must satisfy further as-

Received July 20, 1977. Revised October 25, 1977