

TOPOLOGICALLY COMPLETE SPACES AND PERFECT MAPS

by

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1. Introduction. All spaces considered in this paper are assumed to be completely regular Hausdorff. A space X is called *topologically complete* if it is complete with respect to its finest uniformity. Realcompact spaces and paracompact spaces are topologically complete (cf. [2]). A continuous map f is called *perfect* if it is a closed map and each fiber is compact.

It is known that topological completeness as well as realcompactness is not preserved under perfect maps; this fact was essentially proved by Mrówka [18] and was noted in [3]. To date, the images of realcompact spaces under perfect maps were investigated by several topologists (e.g., Frolík [6], [7], Kenderov [14], Isiwata [11], [13], Blair [1], Dykes [3], [4]), however, with the exception of [3], little seems to be known about topologically complete spaces.

In this paper, we shall obtain characterizations of the images of topologically complete spaces under perfect maps and necessary and sufficient conditions for them to be topologically complete.

In section 2, for convenience, we list certain basic definitions and facts that will be used in the sequel.

In section 3, we introduce the notion of almost uniform structures. This notion is useful for dealing with the perfect images of topologically complete spaces. There are also some “tool” theorems concerning almost uniform structures.

In section 4, almost topologically complete spaces are defined in terms of an almost uniform structure, and we prove that almost topologically complete spaces characterize perfect images of topologically complete spaces. Similarly we can prove the corresponding theorem concerning Frolík’s almost realcompact spaces, and consider the relationship between almost topologically complete spaces and almost realcompact spaces. Furthermore, some properties of almost topologically complete spaces are studied, in particular, it is proved that almost topological completeness is invariant under perfect maps.

In the final section 5, we consider a problem under what conditions an almost