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MONADIC SECOND ORDER LOGIC WITH AN ADDED QUANTIFIER Q

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Introduction

In this paper, we will show that *a monadic second order logic with an added quantifier* Q *is decidable.*

We begin with a description of some known facts concerning the decision problem for the predicate calculus. It was originally shown by L. Löwenheim (1951) that

(1) The monadic fragment of (first order) predicate calculus with equality is decidable.

Simpler proofs of (1) have been given Th. Skolem (1919) and H. Behmann (1922). Likewise there is the following result for the predicate calculus having the Chang quantifier:

(2) The monadic fragment of predicate calculus without equality containing the Chang quantifier is decidable. (A. Mostowski; 1957)

A. Slomson has extended this result further by proving, with the semantic method, that

(3) The monadic fragment of predicate calculus with the Chang quantifier and equality is decidable. (cf. [1])

On the other hand, it is also well-known that

(4) The monadic second order logic is decidable.

The sequential results mentioned above lead us in a natural way to the following "semantic" question: Is the monadic second order logic with the Chang quantifier decidable?

We extend this question to the decision problem formulated "syntactically" below; which turns to have an affirmative answer.

Let L be a monadic second order logic with an added quantifier Q, which will be defined explicitly in § 1. In addition to the usual symbols employed, L has (a) two sorts of unary predicate variables: free and bound, (b) no constants except

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