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CHARACTERIZATIONS OF PARACOMPACTNESS BY INCREASING COVERS AND NORMALITY OF PRODUCT SPACES

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1. Introduction. For paracomparactness and μ-paracompactness (μ—an infinite cardinal), many characterizations have been obtained until now. In particular, for countable paracompactness the following simple characterization is known:

A normal space (a topological space) X is countably paracompact if and only if, for each countable increasing open cover $\{U_n\}$, *there exists a countable closed (open) cover* $\{V_n\}$ *such that* $V_n \subset U_n$ ($\bar{V}_n \subset U_n$) *for each* n (Dowker [4], Ishikawa [5]).

In this paper we shall give new characterizations of paracompactness and μ-paracompactness in terms of "well-ordered increasing cover", and using these characterizations we shall obtain some results with respect to normality of product spaces.

Here a space X is *paracompact* (*μ-paracompact*), if each open cover (with cardinality $\leq \mu$) of X has a locally finite open refinement.

Let λ be an ordinal. We say that a space X has the *property* $P(\lambda)$, if for each open cover $\{U_\alpha | \alpha < \lambda\}$ of X with length λ satisfying

(1) $U_\alpha \subset U_{\alpha+1}$,

(2) $U_\beta = \bigcup_{\alpha<\beta} U_\alpha$ for each limit ordinal $\beta < \lambda$,

there exists an open cover $\{V_{\alpha,n} | \alpha < \lambda, n = 0, 1, 2, \cdots\}$ of X such that

(3) $V_{\alpha,n} \subset V_{\alpha+1,n}$,

(4) $V_{\beta,n} = \bigcup_{\alpha<\beta} V_{\alpha,n}$ for each limit ordinal $\beta < \lambda$,

(5) $\bar{V}_{\alpha,n} \subset V_{\alpha,n+1}$,

(6) $V_{\alpha,n} \subset U_\alpha$.

Our characterizations for paracompactness and μ-paracompactness are as follows:

THEOREM 1.1. *Consider the following statements about a space X:*

(a) *X has the property $P(\lambda)$ for each regular ordinal λ.*

(b) *Each well-ordered increasing open cover $\mathcal{U}$ of X has an open refinement* $\mathcal{V} = \bigcup_{n=0}^{\infty} \mathcal{V}_n$ *such that $\mathcal{V}_n$ is cushioned in $\mathcal{V}_{n+1}$ (in the sense of Michael* [8]) *for*

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