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A GENERALIZATION OF GROUPS WITH A ROOT DATA AND COVERINGS OF THE GROUPS

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0. Introduction

The groups of k-rational points of semi-simple algebraic groups defined over a field k or simple groups of Lie types have a structure of the groups with BN-pairs (Tits system, cf. [2] Chap. IV) or the groups with a root data (due to Bruhat-Tits [3]). On the other hand, Chevalley groups (normal or twisted) over a commutative ring with an identity have also root subgroups but in general, they are neither the groups with BN-pairs nor the groups with a root data. In this note, we treat these groups axiomatically. Namely, we generalize the axioms for the groups with a root data to be able to apply to these groups. Further, we can construct universal convering groups of these groups in the same way as those of R. Steinberg [7].

As for the central extensions of groups of Lie types, C.W. Curtis ([5]) has treated axiomatically and the universal extension of Chevalley groups over a commutative ring has been treated by M. Stein [6] and the result has been generalized to the twisted case by the author [1]. Some of these results can be generalized and simplified by our method.

I would like to express my warm thanks to Professor J. Tits who suggested me the axiomatic treatement of these groups in the same way as those of the groups with a root data and to Professor C.W. Curtis who has invited me to the University of Oregon at the fall term in 1976 and has given many kind discussions about the problem during my stay there.

1. Definition of a group with a root data.

Let E^n be a Euclidean space of dimension n. A subset Φ of E^n is called a *root system* if it satisfies the following properties:

(SR 1) *Φ is a finite subset of E^n such that $0 \notin \Phi$ and $\Phi = -\Phi$ and further Φ spans E^n.*

(SR 2) *For any $\alpha \in \Phi$, let σ_α be the orthogonal transformation of E^n defined by $\sigma_\alpha(x)$*

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