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ON THE DIMENSION OF THE PRODUCT OF TOPOLOGICAL SPACES

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1. Introduction.

As in our previous paper [5], by the covering dimension of a topological space X, dim X in notation, we mean the least integer n such that every finite normal open cover of X is refined by a finite normal open cover of X of order $\leqq n+1$; in case there is no such an integer n, we define dim X to be infinte. This definition was introduced for the case of Tychonoff spaces by M. Katětov [1] and by Yu. Smirnov [9] independently.

In [5] we have proved the following theorem.

"Let Y be a paracompact Hausdorff space which is either locally compact or σ-compact. Then dim $(X\times Y)\leqq \dim X+\dim Y$ for any topological space X."

As is well known, those spaces which are treated conveniently in algebraic topology are CW complexes, and every CW complex is a paracompact Hausdorff space which is a countable union of locally compact closed subsets but which is neither locally compact nor σ-compact in general. Thus, the above theorem is not applicable to the case where Y is a CW complex.

The purpose of this paper is to extend the theorem mentioned above so that it may be applied to the case with Y being a CW complex, by establishing the following theorems.

THEOREM 1. *Let Y be a paracompact Hausdorff space which is a countable union of locally compact closed subspaces. Then for any topological space X we have*

$$\dim(X\times Y)\leqq \dim X+\dim Y.$$

THEOREM 2. *Let X be a topological space and Y a CW complex. Then*

$$\dim(X\times Y)=\dim X+\dim Y.$$

THEOREM 3. *Let X be a topological space with* $\dim X=1$ *and Y a paracompact*

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