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ASYMPTOTIC BEHAVIORS FOR MULTIDIMENSIONAL KIRCHHOFF EQUATIONS

By

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1. Introduction

In the previous paper [8] we discussed the Cauchy problem for Kirchhoff equation in multidimensional spaces and obtained the time global solutions to the Cauchy problem under the assumption that the initial data satisfy an integrable condition. In this paper under the same integrable conditions we shall investigate the asymptotic behaviors concerning $t \to \pm$ for the following Kirchhoff equation

$$u_{tt}(t,x) - (1 - \varepsilon(Au(t), u(t))_{L^2})Au(t,x) = 0, \quad t \in R,\ x \in R^n, \tag{1.1}$$

where $A = \sum_{j,k=1}^{n} \frac{\partial}{\partial x_j} a_{jk}(x) \frac{\partial}{\partial x_k}$ and ε is a positive constant. We assume that $-A$ and $H = \sqrt{-A}$ are non negative definite selfadjoint operators in $L^2(R^n)$. Denote by $D(H) = \{u \in L^2(R^n); Hu \in L^2(R^n)\}$ the definition domain of H. For $(f,g) \in D(H^{(2+k)/2}) \times D(H^{k/2})$, $k \ge 0$ and j a non negative integer, we define

$$\begin{aligned} G_{k,j}(H,f,g,t) = {} & |(e^{itH}H^{2+k}f, \langle t\rangle^j f)| + |(e^{itH}H^{1+k}f, \langle t\rangle^j g)| \\ & + |(e^{itH}H^k g, \langle t\rangle^j g)| \end{aligned} \tag{1.2}$$

and

$$\|(f,g)\|_{Y_{k,j}(H)} = \int_{-\infty}^{\infty} G_{k,j}(H,f,g,t)\,dt$$

where $\langle t\rangle = \sqrt{1+t^2}$ and $(\cdot,\cdot)$ stands for an inner product of $L^2(R^n)$. Denote by $Y_{k,j}(H)$ the set of functions $(f,g) \in D(H^{(k+2)/2}) \times D(H^{k/2})$ satisfying $\|(f,g)\|_{Y_{k,j}(H)} < \infty$. For simplicity we denote $G_{k,0}(H,f,g,t)$ and $Y_{k,0}(H)$ by $G_k(H,f,g,t)$ and $Y_k(H)$ respectively.

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