Poisson bracket under mappings

Dedicated to Prof. Yoshie Katsurada on her sixtieth birthday

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1. Introduction

The main purpose of this paper is to investigate the behavior of Poisson bracket under mappings in terms of differential geometry. The formalism of Poisson bracket relates with the problem in quantizing of dynamical systems. Hence this formalism plays an important role in quantum physical theory. The basic concepts in this paper are given in the literatures by R. Hermann [1], [2].

2. The Poisson bracket defined by a closed 2-differential form

Let M be a differentiable manifold, ω a given closed 2-differential form on M, i.e.,

$$\mathrm{d}\omega = 0\,, \tag{2.1}$$

where d denotes the exterior derivative.

Let M_p be the tangent space to M at p, then a vector $v \in M_p$ is said to be a characteristic vector of ω if

$$v \lrcorner\, \omega = 0 \quad \text{i.e.,} \quad \omega(v, M_p) = 0\,. \tag{2.2}$$

The set $C_p(\omega)$ of all these characteristic vectors forms a subspace of M_p.

The following theorem is shown by R. Hermann ([1], 122–123).

THEOREM 2.1. *Let $v_1, \cdots, v_m$ be a basis for M_p such that $v_1, \cdots, v_n$ form a basis for $C_p(\omega)$; $\omega_1, \cdots, \omega_m$ the dual basis of $v_1, \cdots, v_m$ i.e.,*

$$\omega_i(v_j) = \delta_{ij} \qquad \textit{for} \quad 1 \leq i, j \leq m\,, \tag{2.3}$$

then ω is written at p as follows:

$$\omega = \sum_{i,j>n} a^{ij}\omega_i \wedge \omega_j\,, \quad \det_{n<i,j\leq m}(a^{ij}) \neq 0\,. \tag{2.4}$$

Let $F(M)$ be the ring of C^∞ real valued functions on M, then a function $f \in F(M)$ is said to be an integral of the characteristics of ω if