Notes on the balayaged measure on the Kuramochi boundary*)

Dedicated to Professor Yoshie Katsurada on her 60th birthday

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1. Let p^{μ}, p^{ν} be two Green potentials on a hyperbolic Riemann surface R. Let G be an open set in R. It is well-known that if $p^{\mu}=p^{\nu}+h$ on G for some harmonic function h on G, then the restriction of μ on G equals the restriction of ν on G.

In this paper, we shall prove a similar result to the above is also valid for Kuramochi's potentails and an open set in the Kuramochi compactification R_N^* of R (Theorem 1). As applications, we shall prove the followings: (a) The support of the canonical measure associated with $\tilde{g}_b$ for a non-minimal Kuramochi boundary point b is contained in the closure of the set of all non-minimal Kuramochi boundary points (Theorem 2). As for a non-minimal Martin boundary point, T. Ikegami [2] had obtained an analogous result to (a). (b) Let K be a compact set in $R_0^*=R_N^*-K_0$ (K_0 is a closed disk in R) and $\tilde{C}$ be the Kuramochi capacity on R_0^*. If we denote by $\mathrm{Int}(K)$ the set of all interior points of K in R_0^*, then we have $\tilde{C}(K)=\tilde{C}(K-\mathrm{Int}(K))$ (Theorem 3).

2. Let R be a hyperbolic Riemann surface. We shall use the same notation as in [1], for instance, $\tilde{g}_b$, $\tilde{p}^{\mu}$, f^F, R_N^*, $\varDelta_N$ etc. For a subset A of R, we denote by ∂A the relative boundary of A in R and by $\bar{A}$ the closure of A in R_N^*. The Kuramochi boundary $\varDelta_N$ is decomposed into two mutually disjoint parts: the minimal part $\varDelta_1$ and the non-minimal part $\varDelta_0$. By a measure μ on R_0^*, we always mean a positive measure μ on R_N^* such that $\mu(K_0)=0$. For a measure μ on R_0^*, we denote by $S\mu$ the support of μ and by $\mu|E$ the restriction of μ on a Borel set E in R_N^*. If a measure μ on R_0^* satisfies $\mu(\varDelta_0)=0$, then it is called *canonical*. It is known that if μ is a measure on R_0^*, then there exists a unique canonical measure ν such that $\tilde{p}^{\mu}=\tilde{p}^{\nu}$. For a closed set F in R and measure μ on R_0^*, we denote by μ_F the canonical associated measure with $\tilde{p}^{\mu}{}_{\tilde{F}}$. We note that $S\mu_F\subset\bar{F}$.

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