Surfaces with vanishing normal curvature

Dedicate to Professor Yoshie Katsurada on her 60th birthday

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§1. Introduction.

The normal curvature of a submanifold is defined by the square of the length of the curvature form of the connection in the normal bundle (cf [6]). The minimal index (M-index) at a point of a submanifold is defined by the dimension of the linear space of all second fundamental forms with vanishing trace (cf [8]). In this paper we prove the following proposition:

PROPOSITION. *Let M be a compact connected surface with positive Gaussian curvature G isometrically immersed in a $(2+p)$-dimensional space form N of curvature c. If M is non-minimal and the mean curvature vector H is parallel in the normal bundle and the normal curvature vanishes identically, then M is a totally umbilical surface with M-index 0. Especially if N is euclidean then M is a sphere in a 3-dimensional linear subspace of N.*

Without the assumption that H is parallel the same result holds under the assumption that H never vanishes and $H/\|H\|$ is parallel, if G is constant and c is non-positive, or if the Lipschitz-Killing curvature corresponding to $H/\|H\|$ is constant.

The proof is based on the Laplacian of the length of the second fundamental form (cf [3]). In §2 we recall the connection in the normal bundle and obtain a formula similar to one essenlially used in [6] (cf REMARK 2). In §3 we prove that M is of M-index 0. In §4 we make use of a classical method in the theory of Weingarten surfaces and show that M is pseudo-umbilical and prove the proposition.

§2. Preliminaries.

Let ι be an isometric immersion of an n-dimensional Riemannian manifold M in an $(n+p)$-dimensional space form N with curvature c. We shall make use of the following convention of the range of indices:

$$1\leqq A, B, C, \cdots \leqq n+p\,;\ 1\leqq i, j, k, \cdots \leqq n\,;$$
$$n+1\leqq \alpha, \beta, \gamma, \cdots \leqq n+p\,;\ n+2\leqq r, s, t, \cdots \leqq n+p\,.$$