On some commutor theorems of rings

Dedicated to Professor Yoshie Katsurada on her 60th birthday

By Kozo SUGANO

Introduction

In their papers [7] and [4], Hattori and Endo-Watanabe proved that in a central separable C-algebra, there exists a one to one correspondence of the class of semisimple C-subalgebras to itself (See Theorem 3.5 [7] and Theorem 4.2 [4]). The author tried to extend this theorem to the case of separable extension, and obtained a partial extension of their theorem (See §2). Let $\Lambda|\Gamma$ be a ring extension with ${}_{\Lambda}\Lambda\otimes_{\Gamma}\Lambda_{\Lambda}<\oplus_{\Lambda}(\Lambda\oplus\cdots\oplus\Lambda)_{\Lambda}$ (we call this extension H-separable extension), and denote $\mathfrak{B}_{l}=\{B|B\supset\Gamma,\ {}_{B}B_{\Gamma}<\oplus {}_{B}\Lambda_{\Gamma}$ and $B\otimes_{\Gamma}\Lambda\rightarrow\Lambda$ splits$\}$, $\mathfrak{D}_{l}=\{D|C\subset D,\ {}_{D}D<\oplus_{D}\Delta$ and $D\otimes_{C}\Delta\rightarrow\Delta$ splits$\}$, $\mathfrak{B}=\{B|$separable extension of Γ, ${}_{B}B_{B}<\oplus_{B}\Lambda_{B}\}$ and $\mathfrak{D}=\{D|$separable C-subalgebras of $\Delta\}$. In §0 we state some important properties of H-separable extension which have been obtained already for convenience to readers. In §1 we shall prove that there exist one to one correspondences between $\mathfrak{B}_{l}$ and $\mathfrak{D}_{l}$ and between $\mathfrak{B}$ and $\mathfrak{D}$. The latter correspondence has been proved by the same author under the additional condition that Λ is left or right Γ-$f.g.$ projective. In §2 we shall prove that B in $\mathfrak{B}_{l}$ is left (resp. right) semisimple over Γ, $D=V_{\Lambda}(B)$ is right (resp. left) semisimple over C under the condition that Λ is left Γ-$f.g.$ projective and a C-generator. In §3 we shall give an example of separable extension which is not a Frobenius extension.

0. Preliminaries

All rings in this paper shall be assumed to have unities and all subrings have the same identities as the over rings. First, we shall recall the definitions. Let Λ be a ring and Γ a subring of Λ, C the center of Λ, $\Delta=V_{\Lambda}(\Gamma)=\{x\in\Lambda|xr=rx$ for every $r\in\Gamma\}$.

DEFINITION. Λ is a separable extension of Γ if the map π: $\Lambda\otimes_{\Gamma}\Lambda\rightarrow\Lambda$ defined by $\pi(x\otimes y)=xy$ splits as $\Lambda-\Lambda$-map.

DEFINITION. Λ is an H-separable extension of Γ if $\Lambda\otimes_{\Gamma}\Lambda$ is $\Lambda-\Lambda$-isomorphic to a $\Lambda-\Lambda$-direct summand of a finite direct sum of copies of Λ.

DEFINITION. Λ is a left semisimple extension of Γ if every left Λ-module is (Λ, Γ)-projective, or equivalently, if every left Λ-module is (Λ, Γ)-