On a K-space with certain conditions

Dedicated to Professor Yoshie Katsurada on her 60th birthday

By Hidemaro KÔJYÔ

§0. Introduction.

Recently, K. Takamatsu and Y. Watanabe [2][1] proved that a conformally flat K-space is locally symmetric.

The purpose of the present paper is to investigate the analogous problems in a K-space with $C^{h}{}_{ijk;h}=0$. In §1, we shall give some relations in a K-space to use latter. §2 is devoted to give some results in a K-space with $C^{h}{}_{ijk;h}=0$.

The author likes to express his sincere thanks to Prof. Y. Katsurada and Dr. T. Nagai who gave me many valuable suggestions and constant guidances.

§1. Preliminaries.

Let M^n be an n-dimensional $(n=2m>2)$ almost Hermitian manifold with Hermitian structure $(F_j{}^i, g_{ij})$, i.e. with an almost complex structure tensor $F_j{}^i$ and a positive definite Riemannian metric g_{ij} satisfying

$$F_j{}^i F_i{}^k = -\delta_j{}^k \tag{1.1}$$

$$g_{ab} F_i{}^a F_j{}^b = g_{ij}\,, \tag{1.2}$$

where δ_j^k is the Kronecker's delta.

If an almost Hermitian structure satisfies

$$F_{ij;k} + F_{ik;j} = 0 \qquad (F_{ij} = g_{hj} F_i{}^h) \ [3], \tag{1.3}$$

where the symbol " ; " denotes the operator differentiation with respect to the Riemann connection determined by g_{ij}, then the manifold is called a K-space.

From (1.1), (1.2) and (1.3), it follows that

$$F_{ij} = -F_{ji}\,, \qquad F_i{}^j{}_{;j} = 0\,. \tag{1.4}$$

Let $R^i{}_{jkl}$, $R_{jk} = R^i{}_{jki}$ and $R = g^{ij} R_{ij}$ be the Riemannian curvature tensor, the Ricci tensor and the scalar curvature, respectively. Applying the Ricci's

---

1) Numbers in brackets refer to the references at the end of the paper.