A NOTE ON PRIMITIVE EXTENSIONS OF RANK 3 OF ALTERNATING GROUPS

By

Shiro IWASAKI

1. T. Tsuzuku determined (the degrees of) the primitive extensions of rank 3 of symmetric groups ([4]). In this note we take up alternating groups instead of symmetric groups and prove the following theorem.

Theorem 1. *Let A_n be the alternating group of degree n. If A_n has a primitive extension of rank* 3, *then* $n=1$, 3, 5 *or* 7.

Since our proof is quite simlar to Tsuzuku's paper [4], we use the same notations as those in [4] and give a proof in outline only.

S_n: The symmetric group of degree n.

A_n: The alternating group of degree n (on a set Γ).

G: A primitive extension of rank 3 of A_n on a set $\Omega=\{0, 1, 2, \cdots, n, \tilde{1}, \tilde{2}, \cdots, \tilde{m}\}$ which consists of $1+n+m$ letters.

H: The stabilizer G_0 of a letter, say 0, of Ω. The orbits of H are denoted by $\Delta_0=\{0\}$, $\Delta_1=\{1, 2, \cdots, n\}$ and $\Delta_2=\{\tilde{1}, \tilde{2}, \cdots, \tilde{m}\}$ and the group (H, Δ_1) is isomorphic to (A_n, Γ).

L: The stabilizer of the subset $\{0, \tilde{1}\}$ of Ω.

$|X|$: The number of elements in a set X.

2. *Proof of Theorem 1.* Clearly A_2 does not have a primitive extension of rank 3 and so $n\neq 2$. In the following we assume that $n\neq 1$, 3, 5 and 7. By assumption, the group (A_n, Γ) is isomorphic to (H, Δ_1) and $|L|$ is equal to $\frac{n!}{2m}$. According to a theorem of Manning ([4], 2. Prop. 1), $|L|$ is divisible by $\frac{(n-2)!}{2}$ and $\frac{(n-1)!}{2}>|L|\geqq\frac{(n-2)!}{2}$.

I. The case $|L|>\frac{(n-2)!}{2}$ *and L is transitive on* Δ_1.

If L is a primitive subgroup of (H, Δ_1), then, in the same way as 3. I in [4], $2n(n-1)>\left[\frac{n+1}{2}\right]!$ and so we have $n=10$, 9, 8, 6, or 4. In case $n=10$, 9 or 8, by a theorem of Jordan ([5], th. 13. 9), L is either A_n or S_n and this is a contradiction. For the cases $n=6$ or 4, and also for the case