

ON INTRINSIC THEORIES IN THE MANIFOLD OF SURFACE-ELEMENTS OF HIGHER ORDER

By

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Introduction. It is well known that the space in which a measure of a hypersurface: $x^i = x^i(u^1, u^2, \dots, u^{n-1})$, $i = 1, 2, \dots, n$ is given by the $(n-1)$ -ple integral: $\int_{(n-1)} F(x^i, \partial x^i / \partial u^a) du^1 \dots du^{n-1}$ is called a CARTAN space. As it is shown by CARTAN, this space is to be regarded as a manifold of hyperplane-elements $(x^i, \partial x^i / \partial u^a)$. The geometry of CARTAN space were discussed by E. CARTAN [1]⁽¹⁾ and L. BERWALD [6][7] at large. Thereafter, T. OHKUBO [9] and the present author [10][11] extended this theory to the $(n-1)$ -ple integral of higher order of special forms. Recently, the present author [12] have established a geometry of an $(n-1)$ -ple integral of the second order in general form, but the space in which the theories are discussed was regarded as a manifold of hypersurface-elements of the third order. On the other hand the theory of K -spreads in an n -dimensional manifold which are concerned with a system of partial differential equations of the second order was studied at first by J. DOUGLAS, and the theory was treated in the manifold of all K -dimensional surface-elements of order 1. Thereafter A. KAWAGUCHI and H. HOMBU [5] studied the theory of K -spreads of the m -th order ($m \geq 2$), and the manifold of all K -dimensional surface-elements of the $(m-1)$ -th order was based in this case. In this paper we aim to establish the foundation of differential geometries in the manifold of K -dimensional surface-elements of higher order under the transformation group of the surface-elements which is deduced from the groups of arbitrary transformations of coordinates and parameters, and treat of the geometry of multiple integral of higher order in detail.

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§ 1. The manifold $F_n^{(m)}$ and notations. In an n -dimensional space X_n with point coordinates x^1, x^2, \dots, x^n a K -dimensional surface is defined analytically by the parametric equations

(1) Numbers in brackets refer to the references at the end of the paper.