ON PRIMARY LATTICES

By

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In my previous short note concerning the lattice of all subgroups of a finite abelian group, I have introduced new sorts of lattices[1]. I have called namely a modular lattice, whose quotients are always chains or sublattices with no proper neutral element, *primary* (primär) and the direct union of a finite number of primary lattices *semi-primary* (halb-primär). They enable us not solely to illustrate the intermediate relations between all subgroups of an ordinary abelian group, but further to characterize the lattice of submodules in a module with finite bases, where the ring of operators is primary and uniserial in Köthe's sense[2]. We obtain in this way an extension of the well known fact, that an indecomposable complemented modular lattice of finite dimension $n \geqq 4$ characterizes completely the lattice of all linear subspaces of an n-dimensional projective space, in a direction different from that of J. v. Neumann's continuous geometry. As we assume no preliminary knowledge about modular lattices, we shall deal with them in part I. All properties of primary or semi-primary lattices, which are either characteristic for them or indispensable for the later developement, shall be treated in part 2. As we deal with only the finite-dimensional case, their topological aspect was not considered. The remaining parts concern chiefly with the generalization of von Staudt's algebra of throws, to attain the main theorem, that every primary lattice with $m_h \geqq 4$ is isomorphic with a lattice of submodules in a module of the above mentioned sort[3].

Finally I must tender my hea[illegible]hanks to Mr. Nakayama, who has been kind enough to give me [illegible] useful remarks.

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(1) Über modulare Verbände, welche die Untergruppen einer endlichen abelschen Gruppe bilden. I. Proc. of the Imp. Acad. Vol. XIX. No. 9, 1943.

(2) G. Köthe, Verallgemeinerte Abelsche Gruppen mit hyperkomplexem Operatorenring. Math. Zeitschrift. 39, 1935.

(3) For the meaning of the condition $m_h \geqq 4$ refer the part 2 of this paper.