ON THE THEORY OF A RHEONOMIC CARTAN SPACE

By

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Introduction.

A CARTAN space having its origin in the CARTAN's paper "*Les espace de métriques fondés sur la notion d'aire*" was developed by many people, especially, L. BERWALD.[1] In the present paper we attempt to build up the geometry in this space from standpoint of the rheonomic theory. In this space, its $(n-1)$-dimensional area is assumed to be given *a priori* in such a way that it depends on a variation of time. The geometrical quantities of this space depend on x^a, t, u_a, u_0. If the time-area is independent of u_0, then in every moment this space reduces to a CARTAN space in ordinary sense, i.e. then this area is nothing but that of a CARTAN space. Since u_0 may be interpreted as a velocity of a small piece of hypersurface-element, we shall call u_0 *a velocity of the hypersurface-element*. As u_0 is not invariant under a rheonomic transformation, we introduce a invariant parameter v in place of u_0. This parameter v plays an important rôle in our theory. The form of fundamental function $L(x^a, t, u_a, u_0)$ is rewritten in $G(x^a, t, u_a, v)$ which is homogenous of degree one in u_a and lets us decide the base connection, the connection-parameters, the curvature tensors and identities of BIANCHI in our space.

§ 1. Fundamental function.

In an n-dimensional rheonomic manifold X_n with coordinates x^i, t, we consider a rheonomic hypersurface X_{n-1} given by

$$x^a = x^a(v^1, v^2, \cdots, v^{n-1}, t) \qquad a = 1, \cdots, n \tag{1.1}$$

and suppose that its measure of $(n-1)$-dimensional time-area in a

(1) L. BERWALD: Über die n-dimensionalen CARTANschen Räume und eine Normalform der zweiten Variation eines $(n-1)$-fachen Oberflächenintegrals, Acta Mathematica, 71 (1939), 191-248.