ON THE ORTHOGONAL EXPANSION OF THE BOOLEAN POLYNOMIAL AND ITS APPLICATIONS I

By

Yoemon SAMPEI

Introduction.

The aim of this paper is twofold: to establish orthogonal expansion as a convenient tool in the theory of Boolean algebra; and to render it useful in discussions concerning the structure of the system of mathematical logic particularly in the intrinsic meaning of quantifiers.

In Chapter I, we deal mainly with the orthogonal expansions of propositional polynomials, which are somewhat different from the conjunctive normalform and the disjunctive normalform of logical formulas(1) and are much more convenient to applications than them. Incidentally, our conclusion will be that any (generalized) truth function can be constructed by five operations: logical sum, logical product, negation, universal quantifier and existensive quantifier. Though our discussion is conducted with propositions, yet it should be made clear that the same procedure can be followed with Boolean algebra.

In Chapter II, we consider the structure of the system of mathematical logic. To understand this, let us observe the following fact.

Define the universal quantifier and the existensive quantifier by the axioms

e) $$(\forall x)F(x) \supset F(y),$$

f) $$F(y) \supset (\exists x)F(x),$$

as in HILBERT and ACKERMANN's(2).

Replace

1) $$(\forall x)F(x) \quad \text{by} \quad F(1) \vee F(2)$$

and

(1) Cf. HILBERT and ACKERMANN: Grundzüge der theoretischen Logik, 2-te Aufl., 1938, p. 14.
(2) Cf. HILBERT and ACKERMANN, loc. cit., p. 56.