MODULARS ON SEMI-ORDERED LINEAR SPACES I

By

Michiyo MIYAKAWA and Hidegorô NAKANO

In an earlier paper [1], one of the authors defined modulars on linear spaces and discussed their properties: a functional $m(x)$ on a linear space R is said to be a *modular* on R, if

1) $m(0) = 0$;
2) $m(-a) = m(a)$ for every $a \in R$;
3) for any $a \in R$ we can find a positive number α such that
$$m(\alpha a) < +\infty ;$$
4) $m(\xi a) = 0$ for every positive number ξ implies $a = 0$;
5) $\alpha + \beta = 1$, $\alpha, \beta \geqq 0$ implies for every $a, b \in R$
$$m(\alpha a + \beta b) \leqq \alpha m(a) + \beta m(b);$$
6) $m(a) = \sup_{0 \leqq \xi < 1} m(\xi a)$ for every $a \in R$.

For universally continuous semi-ordered linear spaces R, modulars were considered with adding conditions: 7) $|a| \leqq |b|$ implies $m(a) \leqq m(b)$, 8) $|a| \frown |b| = 0$ implies $m(a+b) = m(a) + m(b)$, and 9) $0 \leqq a_\lambda \uparrow_{\lambda \in \Lambda} a$ implies $m(a) = \sup_{\lambda \in \Lambda} m(a_\lambda)$. (cf. [2])

In this paper we shall discuss modulars on lattice ordered linear spaces only with adding condition 7).

§ 1. Modulars on linear spaces

Firstly we shall give a rough sketch of the properties of modulars on linear spaces which are obtained in [1] and [3], and will be used in this paper. Let $m(x)$ $(x \in R)$ be a modular on a linear space R. A linear functional $\bar{x}(x)$ $(x \in R)$ on R is said to be *modular bounded*, if we can find positive numbers α, β such that

$$\alpha \bar{x}(x) \leqq \beta + m(x) \qquad \text{for every } x \in R.$$

The totality of modular bounded linear functionals on R also builds a linear space which will be called the *modular associated space* of R and denoted by $\bar{R}$. For each $\tilde{a} \in \bar{R}$, putting